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THE ELEMENTS OF PHYSICS

VOL. II

ELECTRICITY AND MAGNETISM

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THE

ELEMENTS OF PHYSICS

A COLLEGE TEXT-BOOK

BY

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AND

WILLIAM S. FRANKLIN

IN THREE VOLUMES

VOL. II

ELECTRICITY AND MAGNETISM

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THE ELEMENTS OF PHYSICS.

Volume II.

CHAPTER I.

DISTRIBUTED QUANTITY.*

306.† **A distributed scalar** is a scalar quantity used to specify ‡ the condition or state of a medium. Such a quantity has, in general, a distinct value for each small part of the medium.

Homogeneous or *uniform distribution* occurs when the quantity has the same value for all parts of the medium; otherwise the distribution is said to be *non-homogeneous*. (See Art. 26, Vol. I.)

* Many of the geometrical conceptions arising from the consideration of distributed quantity, particularly of distributed vectors, are of great importance in the theory of electricity and magnetism. A discussion of distributed quantity is therefore given as an introduction to the present volume. The student should, at starting, read as much as possible of this introduction, dropping it whenever any serious difficulty is encountered; to take it up again and again during the progress of his work upon the main portion of the text. Most of the definitions concerning distributed scalars and vectors are seen to be legitimate, *physical continuity being assumed*, and such proofs as do not make use of Cartesian coördinates are rigorous. The introduction of Cartesian coördinates for the purpose of formulating definitions and proving theorems, to be rigorous, must be accompanied by very elaborate proof in each case that the choice of axes is a matter of indifference. Such Cartesian expressions and developments as are herein given are known to be true, but their use here is more of the nature of *exposition* than of *proof*. The student who wishes to read extensively must be familiar with these Cartesian expressions.

† Articles, figures, and equations are numbered consecutively from the beginning of Volume I.

‡ *Potential* is a distributed scalar which *does not* refer to the state of a medium at each point; it is accordingly highly fictitious and of limited usefulness.

307. Examples of distributed scalars. — The temperature at each point of a body, the hydrostatic pressure at each point of a fluid, the density, *i.e.* the mass per unit volume, at each point of a body, the electric charge per unit volume (volume density of charge) at each point in a charged region, etc., are distributed scalars. The pressure in the atmosphere and the pressure in a heavy liquid are examples of non-homogeneously distributed hydrostatic pressure. The atmosphere is an example of a non-homogeneously distributed density.

308. The volume integral of a distributed scalar. — Let V be the density of a body at a point; then $V \cdot \Delta\tau$ is the mass of material in the volume $\Delta\tau$ at the point and $M = \sum V \cdot \Delta\tau$,* or

$$M = \int V \cdot d\tau. \tag{180}$$

In this equation M is the total mass of the body, in the region throughout which the summation is extended. This summation, $\int V \cdot d\tau$, is called the *volume integral* of the distributed scalar V. The significance of the volume integral of a distributed scalar is not in every case so evident as in the case of density. If V is the volume density of electric charge, then $\int V \cdot d\tau$ is the total electric charge in the region throughout which the summation is extended; if V is the energy per unit volume in an electric field, in a magnetic field, in a strained solid, or in a moving liquid, then $\int V \cdot d\tau$ is the total energy in the region of summation.

309. The gradient of a distributed scalar. — Let V be the value of a distributed scalar at a point p, and $V + \Delta V$ its value at an adjacent point distant Δx from p; then $\dfrac{\Delta V}{\Delta x} = X$ is called

* The student of physics must think in every case of an integration as a summation; the symbols Σ and $\int$ have to him the same meaning.

the *gradient* of V in the direction of Δx. We may therefore write

$$X = \frac{dV}{dx},$$

and similarly
$$Y = \frac{dV}{dy}, \tag{181}$$

$$Z = \frac{dV}{dz}.$$

X, Y, and Z are the components of a definite* vector called the *resultant gradient*, or simply the *gradient* of V at the point p. The gradient of a distributed scalar is thus a distributed vector.

310. A distributed vector is a vector quantity used to specify† the condition or state of a medium. Such a quantity has, in general, a distinct value for each small part of the medium.

Homogeneous or *uniform* distribution occurs when the vector has the same magnitude and the same direction for all parts of the medium, otherwise the distribution is said to be *non-homogeneous*.

311. Examples of distributed vectors.—(1) *Fluid motion:* (*a*) The velocity at each point of a moving fluid is a distributed vector. (*b*) In certain cases of fluid motion each small portion of the fluid is rotating at a *definite angular velocity about a definite axis* (a vector). This rotatory motion of the small parts of a moving fluid is a distributed vector; it is called *vortex motion.*

(2) *Gravitational field.*—A particle of matter, of mass m, when at a given point in the neighborhood of the earth, for example, is acted upon by a force $F = mg$ (Equation 16). The

* To prove this, it is sufficient to show that $\left(\frac{dV}{dx}\right)^2 + \left(\frac{dV}{dy}\right)^2 + \left(\frac{dV}{dz}\right)^2$ is *invariant; i.e.* that it is a constant which is independent of choice of axes of reference. The foundation of proof is the assumed physical continuity of V.

† A distributed vector which *does not* refer to the state of a medium at each point, is highly fictitious and of limited usefulness. Such, for example, is **Vector Potential.**

quantity g has a definite magnitude and a definite direction (parallel to F) at each point in the neighborhood of the earth. It is called the *intensity* of the gravitational field at the point, and it is a distributed vector. This quantity g has reference, no doubt, to some kind of a state of strain at each point of the region surrounding the earth, due to the presence of the earth. No clue has ever been found as to its exact significance.

(3) *Electric field. Magnetic field.* — The intensity, at each point, of an electric field, or of a magnetic field, is a distributed vector. These quantities have reference, no doubt, to states of a medium. (See Chapter VIII.)

312. Stream lines of a distributed vector. — A line drawn through a region so as to be at each point in the direction of a distributed vector at that point, is called a *stream line* of that distributed vector. The manner of distribution of a vector is clearly represented by the use in imagination of such stream lines. In case of vortex motion these lines are called *vortex lines;* in case of an electric field, a magnetic field, or a gravitational field, they are called "lines of force." The term *stream line* will be used in general statements.

A vortex line is, in accordance with the above definition, a line drawn through a moving fluid so as to be at each point in the direction of the axis about which the fluid at that point is rotating. A familiar example of vortex motion is afforded by smoke rings. The vortex lines in a smoke ring are a bundle of concentric circles.

313. Permanent and varying states of vector distribution. — A distributed vector of which the value at each point is constant, in magnitude and direction, is said to have a *permanent state* of distribution; otherwise the vector is said to be in a *varying state* of distribution.

Examples. — If an orifice in a large tank of water be suddenly opened, a perceptible time elapses before the jet of water becomes established. During this interval the velocity of the

water at each point is increasing rapidly; after the jet becomes steady, the velocity of the water at each point remains constant in direction and magnitude. The magnetic field in the neighborhood of a moving magnet or in the neighborhood of a moving or changing electric current is in a *varying state.* The electric field in the neighborhood of moving isolated charges is also in a *varying state.*

314. Rate of change of a distributed vector. — Let α (Fig. 144) represent the value at a given instant of a distributed vector at the point p, and let $\alpha + \Delta\alpha$ represent its value after a time interval Δt has elapsed. Then $\dfrac{\Delta\alpha}{\Delta t} = \dot{\alpha}$, the rate of change of α is a vector, associated with the same point p, and its direction is of course that of $\Delta\alpha$. The vector $\dot{\alpha}$ is thus also a distributed vector.

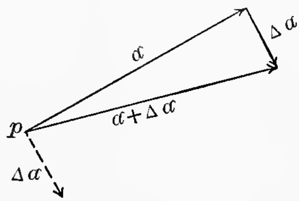

Fig. 144.

Examples. — The rate of change of electric field at a point is parallel to and proportional to the *curl* of magnetic field at the same point. The rate of change of magnetic field at a point is parallel to (but opposite in direction) and proportional to the *curl* of electric field at the point. These are the two laws of electromagnetic induction, so called.

315. The line integral of a distributed vector. — Consider a line pp' (Fig. 145), in the region of a distributed vector. Let Δs be an element of this line, let R represent the value of the distributed vector at the element Δs, and let e be the angle between R and Δs. Then $R \cdot \cos e \cdot \Delta s$ is the scalar part of the product $R \cdot \Delta s$,* and the sum $E = \sum R \cdot \cos e \cdot \Delta s$, or

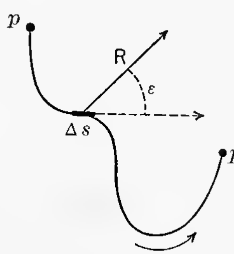

Fig. 145.

$$E = \int R \cdot \cos e \cdot ds, \qquad (182)$$

* This product is part scalar and part vector; so also is the sum $\sum R \cdot \Delta s$, or $\int R \cdot ds$. The vector part of this integral is not, however, of present importance.

is called the *line integral** of the distributed vector R along the line over which the summation is extended. The angle e is reckoned between R and the positive direction of Δs, the positive direction of Δs being the direction in which Δs would be passed over in traveling along the line in a chosen direction. If this chosen direction be changed, $\cos e$ will change sign at each element. Therefore the line integral from p to p' is equal, but opposite in sign, to the line integral from p' to p.

316. Examples of line integrals. — The line integral of the velocity of a fluid along a line is called the *circulation* of the fluid along the line. The line integral of electric field along a line is called the *electromotive force* † along the line. The line integral of magnetic field along a line is called the *magnetomotive force* † along the line.

317. Proposition. — *The line integral of the gradient ‡ of a distributed scalar V, along a line from p to p', is the difference in the values of V at p and at p' respectively.* Proof: Let R (Fig. 145) be the gradient of V; then the resolved part, $R \cdot \cos e$, of R in the direction of Δs is, by Art. 309, the gradient, $\frac{dV}{ds}$, of V in that direction. Therefore $R \cdot \cos e \cdot \Delta s = \frac{dV}{ds}\Delta s = \Delta V$,§

* *Cartesian expression for line integral.* — Let X, Y, and Z be the components of R and dx, dy and dz the components of ds. Then the scalar part of the product $R \cdot ds$ is $X \cdot dx + Y \cdot dy + Z \cdot dz$, as is easily seen by multiplying the vector $X + Y + Z$ by the vector $dx + dy + dz$, and discarding all such products as Xdy, Xdz, Ydx, etc., which are vectors. (See Art. 24, Vol. I.) Therefore equation (182) becomes

$$E = \int (X \cdot dx + Y \cdot dy + Z \cdot dz).$$

† When this line integral is independent of the path from p to p', e. m. f. (or m. m. f.) is called also *difference of electric* (or magnetic) *potential between p and p'*. Thus a difference of electric potential (or of magnetic potential) is always an e. m. f. (or m. m. f.); but an e. m. f. is not always a difference of potential. (See Arts. 317 and 318.)

‡ *A distributed vector.*

§ It follows from this that $X \cdot dx + Y \cdot dy + Z \cdot dz$, which is another expression for $R \cdot \cos e \cdot \Delta s$ (see footnote to Art. 315), is equal to dV; that is, $Xdx + Ydy + Zdz$

so that $\sum R \cdot \cos e \cdot \Delta s = \sum \Delta V$, which is evidently the total change in the value of V from p to p'.

Corollary. — (*a*) The line integral of the gradient of a distributed scalar is the same * for all lines from p to p'; that is, *the line integral of the gradient of a distributed scalar is independent of the path between two points.*

(*b*) If p and p' are the same point, the line pp' is a closed curve. In this case *the line integral of the gradient of a distributed scalar around a closed curve is zero.**

318. Potential of a distributed vector. — It often occurs that a given distributed vector may be considered to be the gradient of a distributed scalar. The distributed scalar is called the *potential* of the given distributed vector.

In order that a given distributed vector may have a potential, it is necessary * and sufficient that the line integral of the vector around all closed curves be zero, or that the line integral be the same along all lines which terminate in the same two points. (Compare Art. 317.) In case of fluid velocity the potential, when it exists, is called *velocity potential.* In case of a gravitational field, the potential is called *gravitational potential.* In case of an electric field, the potential, when it exists, is called *electric potential.* In case of a magnetic field, the potential, when it exists, is called *magnetic potential.* The theory of potential is identical for these four kinds. Magnetic potential

is a *complete differential.* Therefore $\frac{dY}{dx} - \frac{dX}{dy} = 0$, $\frac{dZ}{dy} - \frac{dY}{dz} = 0$, and $\frac{dX}{dz} - \frac{dZ}{dx} = 0$. (See any good treatise on calculus.) These are the conditions which the components X, Y, and Z of a given distributed vector must satisfy at each point in order that that vector may be regarded as a gradient of a distributed scalar. (See Art. 318.)

* The line integral of magnetic field around a closed curve which links n times with a wire carrying an electric current of strength i is $4\pi ni$. Still it is convenient to think of magnetic field as the gradient of a (many valued) distributed scalar. The solid angle ω subtended by a closed *loop*, as seen from *a given point*, is a distributed scalar. Its gradient is of course a distributed vector, and the line integral of this gradient along a closed line which links n times with the given *loop* is $4\pi n$. Solid angle and scalar potential of magnetic field are geometrically identical.

and velocity potential, however, present some aspects which do not occur in electric potential or gravitational potential. See footnote to this article.

319. Significance of electric, magnetic, and gravitational potential.—Let R (Fig. 145) be the intensity at the element Δs of an electric field, then QR is the *force* parallel to R acting upon an electric charge Q at the element, and $QR \cos e$ is the component of this force in the direction of Δs, so that $QR \cdot \cos e \cdot \Delta s$ is the work ΔW done by the field upon the charge Q, as Q is moved along Δs. Therefore

$$W = \sum QR \cdot \cos e \cdot \Delta s = Q \sum R \cdot \cos e \cdot \Delta s$$

is the total work done upon Q as it is carried along the line from p to p'. But $\sum R \cdot \cos e \cdot \Delta s$ is the difference of the values of the electric potential at p and p', so that *the difference of electric potentials at two points is the work per unit charge done by the field when a charge is carried from one point to the other.* Similar statements hold for magnetic potential and magnetic pole, and for gravitational potential and matter.

320. The surface integral of a distributed vector.—Let A and B (Fig. 146) be the points of intersection of a closed curve (a loop) with the plane of the paper, and let the line AB represent a diaphragm to this loop. Let ΔS be the area of a small part of this diaphragm, let R represent the value, at the element ΔS, of a distributed vector, and let e be the angle between R and the normal to ΔS, this normal being drawn always to the same side of the diaphragm. Then $R \cdot \cos e \cdot \Delta S$ is the scalar part of the product $R \cdot \Delta S$,* and the sum

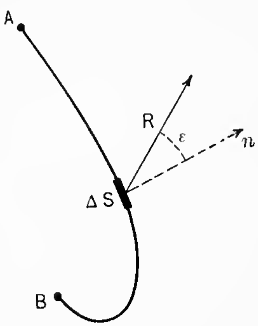

Fig. 146

* This product is part scalar and part vector, so also is the sum $\sum R \cdot \Delta S$, or $\int R \cdot dS$. The vector part of this integral is not, however, of present importance.

$$N = \sum R \cdot \cos c \cdot \Delta S,$$

or
$$N = \int R \cdot \cos c \cdot dS, \tag{183}$$

is called the surface integral of the distributed vector R over that portion of the diaphragm over which the summation is extended. In case the normal be drawn to the other side of the diaphragm, cos c will be everywhere changed in sign, and the surface integral, retaining its numerical value, will be changed in sign. In case of the integration over a closed surface, the normal is understood to be drawn towards the interior always.

The surface integral of fluid velocity over a surface is the *flux* of fluid through the surface in *cubic centimeters per second.* It is convenient to speak of the surface integral of any distributed vector over a given surface as the *flux* of that vector through the surface. In case of a closed surface, we speak of the flux of a vector *into* or *out of* the region bounded by the surface. Thus we speak of flux in the case of an electric field, flux in the case of a magnetic field, etc.

321. Convergence of a distributed vector. — Consider a small region of volume $\Delta\tau$ in the neighborhood of a point p. Let ΔN be the flux of a distributed vector R in this region. Then the ratio $\frac{\Delta N}{\Delta\tau}$ approaches a definite limit as $\Delta\tau$ grows small. This limiting value of $\frac{\Delta N}{\Delta\tau}$ is called the convergence,* ρ, of R at the

* *Cartesian expression for convergence* (exposition). — Consider at a given point p a small cubical region, whose edges are Δx, Δy, and Δz. Let X, Y, and Z be the components of R at p. The flux of R across one face $\Delta y \cdot \Delta z$ is $X\Delta y\Delta z$ *into* the region, and across the other face $\Delta y \cdot \Delta z$ is $\left(X + \frac{dX}{dx}\Delta x\right)\Delta y \cdot \Delta z$ *out* of the region. Therefore the total flux into the region across these two faces is

$$-\frac{dX}{dx}\Delta x \cdot \Delta y \cdot \Delta z.$$

Similar expressions hold for the other pairs of faces, so that the total flux into the region is $-\left(\frac{dX}{dx} + \frac{dY}{dy} + \frac{dZ}{dz}\right)\Delta x \cdot \Delta y \cdot \Delta z$. This quantity divided by the volume $\Delta x \cdot \Delta y \cdot \Delta z$ of the region gives the convergence $\rho = -\left(\frac{dX}{dx} + \frac{dY}{dy} + \frac{dZ}{dz}\right)$ at the point p.

point p. From this definition we have

$$\Delta N = \rho \cdot \Delta\tau, \tag{184}$$

in which ΔN is the flux of a distributed vector R into a region $\Delta\tau$, and ρ is the (mean) convergence of R into that region. The convergence of a distributed vector is, therefore, a distributed scalar having a definite value (often zero) at each point. A negative convergence is sometimes for brevity called a *divergence*.

322. Breaking up of a surface integral over a closed surface into volume elements. — Consider the surface integral of a distributed vector R over a closed surface (normal inwards; see Art. 320). Imagine the region inclosed by this surface to be broken up into a large number of *cells*. Then the surface integral of R over the given closed surface is equal to the sum of the surface integrals of R over the inclosing surfaces of all these cells (normal inwards). For since *any wall which separates two cells is integrated over twice with normals turned oppositely in each case, therefore the only surface integrals left outstanding are those over the portions of the given closed surface.*

Let ΔN be the surface integral of R over one of these cells. Then, from the above, we have $\int R \cdot \cos e \cdot \Delta S = \sum \Delta N$, $\int R \cdot \cos e \cdot \Delta S$ being the surface integral of R over the given closed surface. Now $\Delta N = \rho \cdot \Delta\tau$ from equation (184), where ρ is the convergence of R in the given cell and $\Delta\tau$ the volume of the cell. Therefore $\sum \Delta N = \sum \rho \cdot \Delta\tau = \int \rho \cdot d\tau$, so that

$$\int \rho \cdot d\tau = \int R \cdot \cos e \cdot dS. \tag{185}$$

This gives the following **proposition**: *The volume integral, $\int \rho \cdot \Delta\tau$, of the convergence of a distributed vector R throughout a region is equal to the surface integral, $\int R \cdot \cos e \cdot dS$, of R over the boundary of the region.*

323. Examples of convergence. (1) *Fluid velocity.*—The convergence of fluid velocity at a point is the rate of increase of the density of the fluid at that point. (Principle of continuity in Hydrodynamics.) Imagine a region in an incompressible fluid in which fluid is *annihilated* at the rate ρ *cubic centimeters of fluid per second in each cubic centimeter of the region.* Then the rate of annihilation of fluid in a volume $\Delta\tau$ of the region is $\rho\cdot\Delta\tau$, and this is necessarily equal to the flux of fluid ΔN into the region, so that $\rho\cdot\Delta\tau=\Delta N$ or $\rho=\dfrac{\Delta N}{\Delta\tau}$. (Compare Art. 321.) The *convergence* of the velocity of an incompressible fluid at a point is therefore the rate of annihilation of fluid per unit volume of region in the neighborhood of the point. If fluid is created, then the convergence is negative.

(2) *Electric field.*—The intensity f of an electric field at a point distant r from an electric charge Q is $f=\dfrac{Q}{r^2}$. (See equation 240.) When Q is a *positive* charge, f is by convention considered to be directed *away* from Q. Consider a spherical surface of radius r, area $4\pi r^2$, with its center at Q, then the flux through this surface is $N=-4\pi r^2\cdot f=-4\pi Q$. Dividing this flux, $-4\pi Q$, by the volume τ of the sphere gives the average convergence ρ of f into the sphere. That is $\rho=4\pi\dfrac{Q}{\tau}$, or ρ is equal to -4π times the average volume density of charge in the region. Therefore the convergence of electric field at a point is *minus* 4π times the volume density of charge at the point.

(3) *Magnetic field.*—From the equation $f=\dfrac{m}{r^2}$ (193), expressing the strength of a magnetic field f at a distance r from a pole of strength m, we have a proposition identical to the above for the electric field. Strictly speaking, *magnetic field* has no convergence. It often occurs, however, that we are concerned only with what takes place in the region outside

of a magnet. In this case it is conveniently simple to consider that the magnetic field *diverges* from points distributed throughout one end of the magnet (the north pole of the magnet) and *converges* towards points distributed throughout the other end of the magnet (the south pole of the magnet). This leads to no error so long as one does not attempt to make use of this conception of convergence in the study of what takes place inside of a magnet. In case of the magnetic field due to wires carrying electric currents, the scheme for enabling the application of the conception of convergence is highly artificial, involving as it does the use of *magnetic shells.*

(4) *Gravitational field* at a point is defined by the equation $F = m_1 g$ (see Art. 311). From Newton's law of gravitation, we have $F = k\frac{m_1 m_2}{r^2}$ (see Art. 57, Vol. I.), in which F is the force of attraction of two particles whose masses are m_1 and m_2 and whose distance apart is r. If the force $F = m_1 g$ is the force, $F = k\frac{m_1 m_2}{r^2}$, due to the particle m_2 at a distance r, we have $g = k\frac{m_2}{r^2}$, in which g is the intensity of the gravitational field at a distance r from a particle of mass m_2. Consider a spherical surface: radius r, area $4\pi r^2$, center at m_2. The gravitational flux into this sphere is $N = 4\pi r^2 g = 4\pi k m_2$, which, by the above method for the electric field, shows that *the convergence of gravitational field at a point is equal to $4\pi k$ times the density of matter at that point.*

324. The flux into any region, in the case of the electric field, is equal to the product -4π times the total electric charge in the region. — This statement may be derived by substituting *minus 4π times volume density of charge* for ρ, in equation 185. A similar statement applies to the magnetic field.

Substituting, in like manner, $4\pi k$ *times density of matter* for ρ, in equation 185, we have the statement that *the gravitational flux into any region is equal to $4\pi k$ times the total amount of matter in the region.*

325. Solenoidal vector distribution. — A distributed vector is said to have *solenoidal* distribution in a region throughout which its convergence is zero. The surface integral of such a distributed vector over (or the flux into) any closed surface is zero.

Tube of flow. — Imagine stream lines to be drawn through each point in the periphery of a closed curve or loop. These stream lines form a tubular surface called a *tube of flow* of the given distributed vector.

Consider a number of diaphragms across a tube of flow; *the flux is the same across each.*

Proof. — Any two diaphragms, together with the walls of the tube, constitute a closed surface into which the total flux is zero. The flux across the walls of the tube is zero, therefore the flux into the region across one diaphragm is equal to the flux out of the region across the other diaphragm. A tube of flow is thus *characterized* throughout its length by a certain flux.

Unit tube. — A tube of flow is called a *unit tube* when the flux through it is unity.*

Imagine the region of a distributed vector (solenoidal) to be divided up into unit tubes; then the flux across any surface in the region is equal to the number of these unit tubes which pass through the surface. Each unit tube may be conveniently represented, in imagination, by the single stream line along its axis. Then the flux through any surface in the region will be equal to the number of these lines passing through the surface. In case of the magnetic field (and also of the electric field), the *lines of force* are always thought of as representing each a *unit*

* *Definition of unit flux.* — Let ΔS be a plane area *perpendicular* to the distributed vector R. The flux through ΔS is $R \cdot \Delta S$. When this product has the value unity, the flux through ΔS is called unit flux. For example, in a fluid moving at a velocity of $10 \frac{\text{cm.}}{\text{sec.}}$, the flux across an area of $\frac{1}{10}(\text{cm.})^2$ perpendicular to the velocity is $1 \frac{(\text{cm.})^3}{\text{sec.}}$, *i.e. unit flux.*

tube. The total flux across a surface is then expressed as the number of these lines crossing the surface.

Distributed Vectors having no (Scalar) Potential.

326. Breaking up a line integral around a closed curve into surface elements. — Consider a distributed vector of which the line integral around the closed (heavy) line AB (Fig. 147) is not zero. Let the arrow represent the direction in which this line integral around AB is taken (compare Art. 315). *This line integral around AB is equal to the sum of the line integrals around the various meshes of any network, plane or otherwise, constructed in AB.* The line integrals around the various meshes are to be taken in the same direction as the line integral around AB, as shown by the two or three curled arrows.

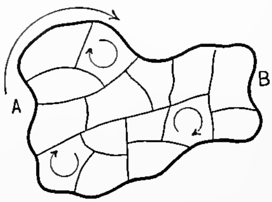

Fig. 147.

Proof. — Any line in the network, not a portion of the heavy line, is integrated over *once in each direction* in integrating around adjoining meshes; therefore (compare Art. 315) only the integrals along the portions of the heavy line are left outstanding.

Let ΔL be the line integral around one of the meshes in Fig. 147. Then, from the above, we have

$$\int R \cdot \cos c \cdot ds = \sum \Delta L, \qquad (186)$$

where $\int R \cdot \cos c \cdot ds$ is the line integral of R around AB.

This breaking up of the line integral around a closed curve into a number of elements, each of which refers to a small portion of a diaphragm (each mesh being a portion of a diaphragm to AB) to the closed curve, shows that the line integral around AB can be expressed as a surface integral over any diaphragm to AB.

326 a. Curl of a distributed vector. — Let ΔL be the line integral (a scalar) of the distributed vector R around one of the meshes of Fig. 147 of which the area is ΔS. The ratio $\frac{\Delta L}{\Delta S}$ approaches a definite limiting value as ΔS approaches zero. This limiting value is the resolved part normal to ΔS of a certain vector C, called the *curl* of the vector R. Let e be the angle between the normal to ΔS and C. Then $\frac{\Delta L}{\Delta S} = C \cdot \cos e$ or $\Delta L = C \cdot \cos e \cdot \Delta S$ which substituted in equation 186, gives

$$\int R \cdot \cos e \cdot \Delta s = \int C \cdot \cos e \cdot \Delta S.$$

That is, *the line integral of a distributed vector around a closed curve, or loop, is equal to the surface integral of its curl over any diaphragm to the loop.*

Examples of curl. — Vortex motion of a fluid is the curl of the fluid velocity. Electric current is the curl of magnetic field. In a changing electric field the rate of change of the intensity of the electric field at a point is proportional to the curl of magnetic field and in a changing magnetic field the rate of change of the intensity of the magnetic field is opposite in direction and proportional to the curl of electric field. This mutual relation of electric and magnetic fields is ordinarily expressed as follows: The electromotive force (line integral of electric field) around any loop is equal and opposite in sign to the rate of change of magnetic flux (surface integral of rate of change of magnetic field) through the loop. A corresponding statement holds for magnetomotive force and electric flux.

326 b. Cartesian expression for curl. — (Exposition.) Consider the value of a distributed vector R in the neighborhood of a point. Let X, Y, and Z be the components of R. Consider the line integral of R around a small

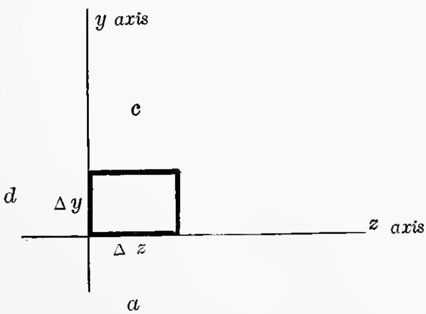

Fig. 148.

square whose sides are Δy and Δz. (Fig. 148.) The line integral of R along the side Δy is $Y \Delta y$, along the side parallel to this the line integral is $-\left(Y + \frac{dY}{dz}\Delta z\right)\Delta y$. The total line integral along these two sides is therefore

$-\frac{dY}{dz}\cdot\Delta z\cdot\Delta y$. Similarly the total line integral along the sides Δz is $+\frac{dZ}{dy}\cdot\Delta y\cdot\Delta z$. So that the line integral around the square is

$$\left(\frac{dZ}{dy}-\frac{dY}{dz}\right)\Delta y\cdot\Delta z,$$

which divided by the area $\Delta y\cdot\Delta z$ of the square gives

$$x\text{-component of curl of } R=\frac{dZ}{dy}-\frac{dY}{dz}.$$

Similarly

$$y\text{-component of curl of } R=\frac{dX}{dz}-\frac{dZ}{dx}$$

$$z\text{-component of curl of } R=\frac{dY}{dx}-\frac{dX}{dy}.$$

326 c. Vector potential. — It can be easily shown that the curl of any distributed vector is a distributed vector which *has no convergence*. The necessary and sufficient condition that a given vector must satisfy in order that it may be looked upon as the curl of some other distributed vector is that it have no convergence. The distributed vector of which a given distributed vector is the curl is called the *vector potential* of the given vector. The condition that a distributed vector may have a scalar potential is that the vector have no curl. Compare Arts. 317, 318, and 326.

CHAPTER II.

MAGNETISM.

327. Magnet; compass; attraction and repulsion.—(*a*) A bar of steel which has been treated in a manner to be described later is called a *magnet*.

(*b*) A magnet suspended horizontally so as to turn freely about a vertical axis places itself, at most places on the earth, so as to point approximately north and south. Such a suspended magnet, usually playing over a divided circle, is called a *compass*. The north pointing end of a magnet is called its *north pole;* the south pointing end, its *south pole*.

(*c*) Like poles of two magnets *repel* each other, unlike poles *attract* each other. A magnet broken in two gives two complete magnets, each having a north pole and a south pole. It is impossible to produce a magnet which has only a north pole or only a south pole.

328. Distributed and concentrated poles.—The *seat* of the attracting or repelling forces in any magnet is distributed over considerable portions of the bar, generally the end portions. This is particularly the case with short, thick bars. In the case of long, slim bars the *seat* of these forces is, ordinarily, more nearly concentrated at the ends of the bar. In the former case the *poles* are said to be *distributed*, in the latter case the *poles* are said to be (approximately) *concentrated*. We shall come later to recognize distinctly the existence of something* which emanates from one pole of a magnet, traverses the surrounding region, and enters the other pole. The *poles* of a magnet are strictly defined as those portions of a magnet from which this

* Magnetic flux, see Art. 505.

emanation takes place; they are concentrated when the portion of the magnet from which this emanation takes place is very small.

329. Axis of a magnet. — The line joining the poles of a magnet is called its *axis*. In case the poles are *distributed*, the axis is still distinct, but the definition in such a case is more complex.

330. Coulomb's law. — The force F with which two concentrated magnetic poles repel one another (attraction is considered a *negative repulsion*) is inversely proportional to the square of the distance d between them. That is

$$F = \frac{k}{d^2}, \tag{187}$$

in which k is a determinate constant for a *given pair of poles*.

331. Strength of pole. — The quantity k of Art. 330, being associated with a *pair* of poles, is not convenient for purposes of numerical specification. It can be shown, however, from Coulomb's law, that there is a certain quantity m associated singly with any given magnetic pole, as follows:

Consider three concentrated magnetic poles, 1, 2, and 3; for example, the north poles of three long, slim magnets. The quantity k, equation (187), has a definite value for *each pair* of these three poles. Let k_{12}, k_{23}, and k_{13} be the values of k associated with the pairs 1 2, 2 3, and 1 3 respectively. We may determine three quantities, m_1, m_2, and m_3 such that

$$\begin{aligned} m_1 m_2 &= k_{12} \\ m_2 m_3 &= k_{23} \\ m_1 m_3 &= k_{13} \end{aligned} \tag{188}$$

These equations give m_1, m_2, and m_3 when k_{12}, k_{23}, and k_{13} are known. By inspection of equations (188) it is evident that m_1 is associated with pole 1; m_2 with pole 2; and m_3 with pole 3.

The quantities m_1, m_2, and m_3 are called the *strengths* of the respective poles. A distributed pole also has a definite *strength*, but in this case the definition is not so simple.

Substituting $m_1 m_2$ for k in equation (187), we have

$$F = \frac{m_1 m_2}{d^2}. \tag{189}$$

Remark. — The reader need not expect to find any one of the various electrical quantities — strength of pole, strength of field, strength of current, resistance, electromotive force, etc. — so directly or so simply connected with the *thing* as to give him any immediate clue to the physical nature of the latter.

332. Algebraic sign of magnetic pole. — The force F in equation (189) is considered *positive* when it is a *repulsion*, and *negative* when it is an *attraction*. Therefore m_1 and m_2 are opposite in sign when one is a north pole and the other a south pole, for F is in this case negative. It is customary to consider a north pole as positive and a south pole as negative.

333. Dimensions of strength of pole. — Writing m^2 for $m_1 m_2$ in equation (189), substituting the dimensions, $\left[\frac{ML}{T^2}\right]$ of F, and solving for m, we have the dimensional equation

$$m = \left[\frac{M^{\frac{1}{2}} L^{\frac{3}{2}}}{T}\right]. \tag{190}$$

C. G. S. unit pole. — The force F in equation (189) being expressed in dynes and d in centimeters, m is said to be expressed in *c. g. s. units*. A *c. g. s. unit pole* is therefore a pole of such strength as to exert a force of one dyne upon an equal pole at a distance of one centimeter. Having no name, the c. g. s. unit pole is often specified in terms of its dimensions; thus 100 gr. $^{\frac{1}{2}}$ cm. $^{\frac{3}{2}}$ sec. $^{-1}$ is read 100 *c. g. s. units pole*.

334. The magnetic moment of a magnet is defined as the product of the strength m of one of its poles into the distance l between its poles; that is,

$$M = ml. \tag{191}$$

A magnet with distributed poles also has a definite *Magnetic Moment;* but in this case the definition is more complex.

335. A magnetic field is a region in which a magnetic pole is acted upon by a force tending to pull it in some direction or other. Thus the region surrounding a wire carrying an electric current, the region surrounding a given magnet, etc., are magnetic fields.

336. Experimental law. Intensity of field at a point. — The force F which acts upon a magnetic pole when it is placed *at a given point* in a magnetic field, is proportional to the strength m of the pole. That is,

$$F = fm, \tag{192}$$

in which f is a determinate constant *for a given point in a magnetic field.* This quantity f, thus associated with the given point, is called the *intensity of the magnetic field at the point.*

337. The direction of a magnetic field at a point. — The force with which a field acts upon a north pole is opposite in direction to the force with which the field at the same point acts upon a south pole. The direction of the force with which the field acts upon a north pole is adopted as the direction of the field.

338. Intensity and direction of field at a distance d from an isolated pole. — A pole m_2 at a distance d from a pole m_1 is acted upon by a force $F = \frac{m_1 m_2}{d^2}$ from equation (189), or $F = \frac{m_1}{d^2} \cdot m_2$. Comparing this with equation (192), we see that

$f = \frac{m_1}{d^2}$, in which f is the intensity of field at m_2 due to m_1. It follows therefore that

$$f = \frac{m}{d^2}, \tag{193}$$

in which f is the intensity of field at a distance d from an isolated pole of strength m. Since two north poles repel each other, the field in the neighborhood of an isolated north pole is directed away from the pole. In the neighborhood of a south pole, on the other hand, the field is directed towards the pole.

339. Dimensions of intensity of field. — Substituting dimensions of $F\left(=\frac{ML}{T^2}\right)$, and dimensions of $m\left(=\frac{M^{\frac{1}{2}}L^{\frac{3}{2}}}{T}\right)$ in (192), and solving for f, we have the dimensional equation

$$f = \left[\frac{M^{\frac{1}{2}}}{L^{\frac{1}{2}}T}\right]. \tag{194}$$

340. C. G. S. unit field. — The force F in equation (192) being expressed in dynes, and m in c. g. s. units, f is expressed in *c. g. s. units.* A c. g. s. unit field is, therefore, a field of such intensity as to exert a force of one dyne upon a c. g. s. unit pole. Having no name for the c. g. s. unit field, it is often specified in terms of its dimensions. Thus 40 gr.$^{\frac{1}{2}}$ cm.$^{-\frac{1}{2}}$ sec.$^{-1}$ is read 40 *c. g. s. units field.*

341. Homogeneous fields. — A magnetic field is said to be *homogeneous* or *uniform* when it has at every point the same direction (and intensity); otherwise it is said to be *non-homogeneous.* The earth's magnetic field is in many places sensibly homogeneous throughout a room. The magnetic field surrounding a wire carrying an electric current, or the magnetic field surrounding a magnet, is non-homogeneous.

342. Representation of a field by means of a line. — The magnetic field at a point, like the velocity of a fluid at a point, may be represented by a line drawn in the direction of the field at the point, and of such length as to represent the intensity of the field at the point to any convenient scale.

343. Superposition of fields. — Consider any two agents which, acting singly, produce magnetic fields whose intensities at a point p are represented by the lines 1 and 2 (Fig. 149), respectively. These two agents acting together produce a magnetic field whose strength at p is represented by the line 3, which is the vector sum of 1 and 2.

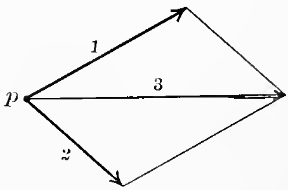

Fig. 149.

344. Resolution of a field into components. — Consider a magnetic field whose strength at a point p is R (Fig. 150). It is often convenient to consider only that part H of this field which falls in a certain direction, a horizontal direction, for example. Thus H is called the horizontal component of R, and V is called the vertical component of R.

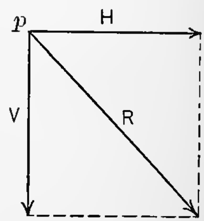

Fig. 150.

345. Behavior of a magnet in a magnetic field. — Consider a magnet of length l, placed in a uniform magnetic field of intensity f, the angle between f and the axis of the magnet being θ, as shown in Fig. 151. The poles of the magnet, being of strengths $+m$ and $-m$, are acted upon by forces $+mf$ and $-mf$, respectively. The torque of each of these forces about o is $mf \cdot \frac{l}{2} \sin\theta$. It follows, therefore, that

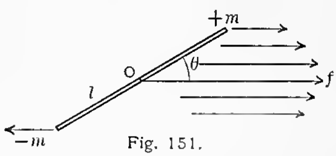

Fig. 151.

$$T = -mlf \sin\theta \tag{195}$$

is the total torque tending to turn the magnet into the direction of the field. The negative sign is chosen since the torque tends to *reduce* θ, which is a positive angle. When $\theta = 0$, this torque is zero, so that a magnet free to turn is in equilibrium only when it points in the direction of the magnetic field in which it is placed.

If θ is very small, then θ may be written for $\sin\theta$ in equation (195), giving

$$T = -mlf \cdot \theta. \tag{196}$$

Comparing this with equation 70, Vol. I., viz., $T = -b\phi$, we see that a magnet in a field, if started, will vibrate to and fro in such a manner that

$$\frac{4\pi^2 K}{\tau^2} = Mf, \tag{197}$$

in which K is the moment of inertia of the magnet, τ is the period of one vibration, and $M (= ml)$ is the magnetic moment of the magnet. Equation (197), depending as it does upon (196), is not true if θ reaches a large value.

346. Gauss's method for measuring the horizontal component H of the earth's magnetic field, and for measuring the magnetic moment of a magnet.

First arrangement. — A large magnet, whose magnetic moment is M, is suspended at the place where H is to be measured, so as to turn freely about a vertical axis, and set vibrating. From equation (197) we have, writing H for f;

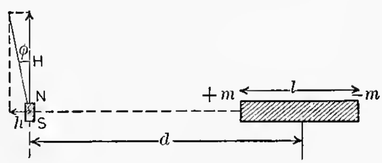

Fig. 152.

$$\frac{4\pi^2 K}{\tau^2} = MH. \tag{i}$$

Second arrangement. — A small magnet NS (Fig. 152) is suspended at the place occupied by the large magnet in the first

arrangement. This small magnet being free to turn, points in the direction of the magnetic field in which it is placed, *i.e.* in the direction of H. The large magnet is now placed with its center at a distance d due magnetic east or west of NS, with its axis pointing towards NS, as shown in Fig. 152. This large magnet then produces at NS a field h at right angles to H. The small magnet now points in the direction of the resultant of H and h, having turned through an angle ϕ. From the diagram (Fig. 152) we have

$$\tan\phi = \frac{h}{H}. \tag{ii a}$$

In case the large magnet is turned so that the field h is at right angles to the deflected magnet NS, as shown in Fig. 153, then we have

$$\sin\phi = \frac{h}{H}. \tag{ii b}$$

From equation 193 we have $\dfrac{-m}{\left(d+\dfrac{l}{2}\right)^2}$ as the expression for the intensity of the field at NS due to the pole $-m$, and $\dfrac{+m}{\left(d-\dfrac{l}{2}\right)^2}$ for the field at NS due to the pole $+m$; so that

$$h = \frac{m}{\left(d-\dfrac{l}{2}\right)^2} - \frac{m}{\left(d+\dfrac{l}{2}\right)^2}. \tag{iii}$$

In this equation, since $\dfrac{l}{2}$ is small compared with d in every practical arrangement of the apparatus, we may write $d^2 - dl$ for $\left(d-\dfrac{l}{2}\right)^2$, and $d^2 + dl$ for $\left(d+\dfrac{l}{2}\right)^2$. This done, we have, after reduction,

$$h = 2\frac{ml}{d^3}\left[\frac{1}{1-\dfrac{l^2}{d^2}}\right]. \tag{iv}$$

Writing M for ml, and substituting this value of h in (ii), we have

$$\frac{M}{H}=\frac{1}{2}d^3\cdot\tan\phi\cdot\left(1-\frac{l^2}{d^2}\right). \tag{v}$$

The large magnet being now placed nearer to NS, say at a distance d', and ϕ' being the angle through which NS is then deflected, we have, in a similar manner,

$$\frac{M}{H}=\frac{1}{2}d'^3\cdot\tan\phi'\cdot\left(1-\frac{l^2}{d'^2}\right). \tag{vi}$$

The uncertain quantity l, which is the distance between the poles of the magnet, may be eliminated from (v) with the help of (vi), giving

$$\frac{M}{H}=\frac{d^5\tan\phi-d_1{}^5\tan\phi_1}{2\,(d^2-d_1{}^2)}. \tag{vii a}$$

In case the large magnet is turned so that the field h is at right angles to NS; then this expression for $\frac{M}{H}$ becomes

$$\frac{M}{H}=\frac{d^5\sin\phi-d_1{}^5\sin\phi_1}{2(d^2-d_1{}^2)}. \tag{vii b}$$

Observations and calculations. — The quantity τ, equation (i), is observed, and K is calculated from the measured mass and dimensions of the magnet, leaving only M and H unknown in (i). The quantities d, d_1, ϕ, and ϕ_1, in equation (vii) are observed, leaving only M and H unknown in (vii). Equations (i) and (vii) then enable the calculation of both M and H. If it is desired to determine the strength of the poles of the large magnet, the quantity l may be approximately measured, whence $m=\frac{M}{l}$. This method * for determining H and M was devised by Gauss.

* For fuller discussion of Gauss's method see A. Gray, Absolute Measurements in Electricity and Magnetism, Vol. II., p. 69.

347. The Kew magnetometer is a form of apparatus, adopted by the Kew Observatory, for performing conveniently the various operations required in carrying out Gauss's method as described above.

It consists of a closed chamber, in which a large magnet is suspended, for the purpose of observing its time of vibration

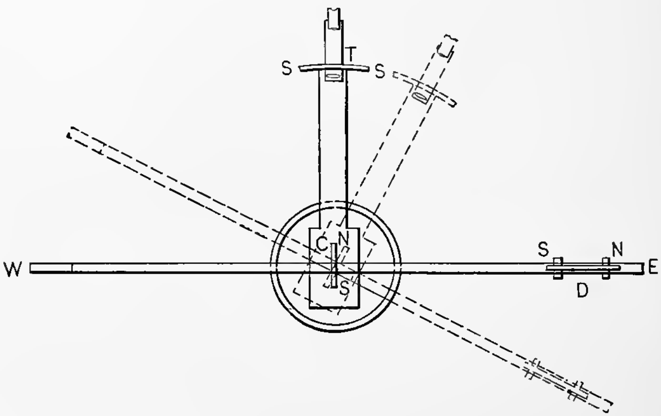

Fig. 153.

(*first arrangement*). This magnet SN, Fig. 153, is then placed upon a rigid bar or scale (EW) extending east and west so as to deflect a small magnet which is now suspended in the chamber (*second arrangement*). This small magnet, Fig. 154, carries a mirror, and for the sake of portability the telescope and scale are attached to the magnetometer itself. To avoid the use of a long scale, the whole apparatus is arranged to turn about a vertical axis, which coincides with the axis of suspension of the small magnet. When a deflection is to be observed, the large magnet is placed upon the long bar, and the whole instrument is turned until the reading of the telescope-mirror-and-scale is the same as before. The angle through which the instrument is thus turned is the required angle of

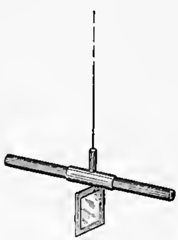

Fig. 154.

deflection, and is read off a divided circle as in a surveyor's transit. Under these conditions, equation (vii *b*) is used, viz.:

$$\frac{M}{H} = \frac{d^5 \sin \phi - d_1^5 \sin \phi_1}{2\,(d^2 - d_1^2)} \tag{198}$$

The reason for the substitution of sin ϕ in the above equation for tan ϕ in equation (vii *a*) of Art. 346 is obvious, since the action of the deflecting magnet is always normal to the axis of the suspended magnet, whatever position the latter may assume.

Figure 155 shows the usual form of the Kew magnetometer.

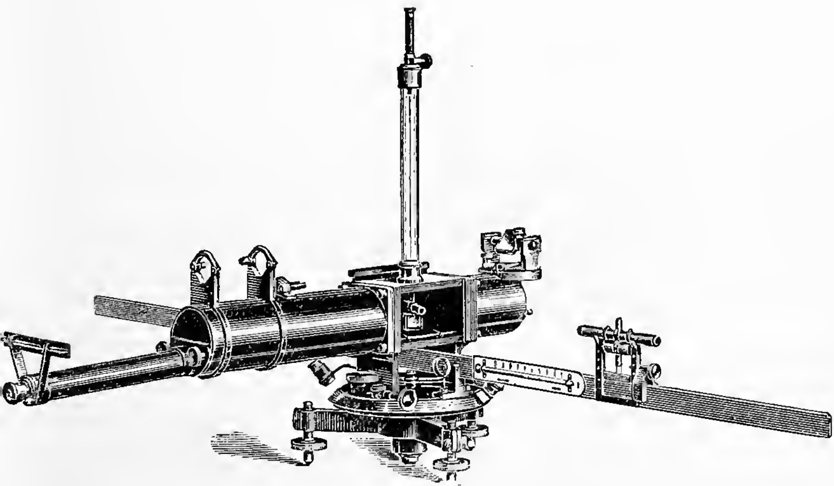

Fig. 155.

348. Lines of force in a magnetic field. — A line drawn through a magnetic field so as to be at each point in the direction of the field at that point is called a *line of force*. When thinking of a magnetic field, the trend of the various lines of force should be kept in mind as serving best to represent all that is geometrically essential in connection with the field.

349. Magnetic figures. — If a pane of glass is placed over a magnet and iron filings dusted over it, the iron filings, becoming

magnetized, tend to arrange themselves in filaments along the lines of force. Slight tapping of the glass facilitates this arrangement of the filings. Figures 156 to 163 are photographic reproductions of magnetic figures obtained in this way.

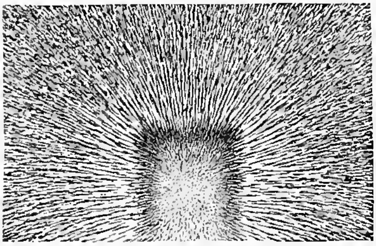

Fig. 156.

Examples. — (*a*) In the neighborhood of an isolated pole the lines of force are straight lines radiating from the pole. Figure 156 is a side view of one end of a magnet, the pole being distributed over that portion of the bar from which lines of force diverge. Figure 157 is an end view of a magnet. Figure 158 shows the nature of the field around an ordinary bar magnet.

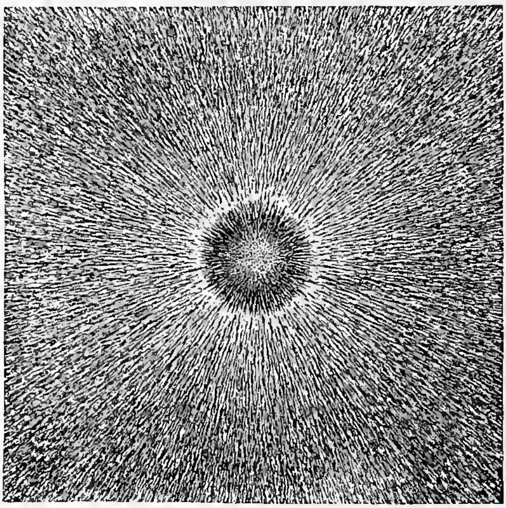

Fig. 157.

(*b*) In the neighborhood of two adjacent opposite poles the lines of force are as shown in Figs. 159 and 160. Figure 159

shows an end view of two magnets side by side. Figure 160 shows a side view of two such poles.

(*c*) In the neighborhood of two adjacent similar poles the

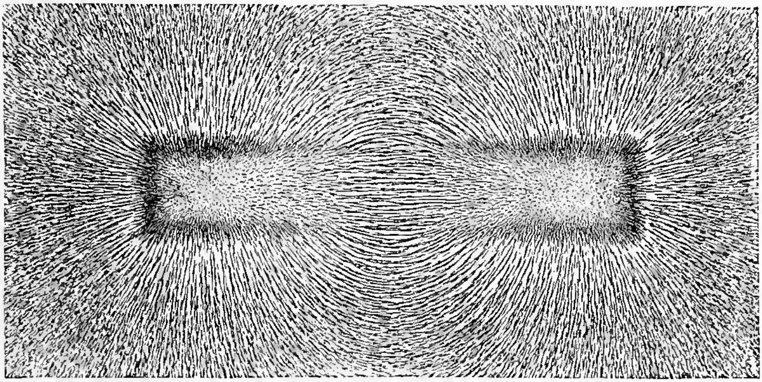

Fig. 158.

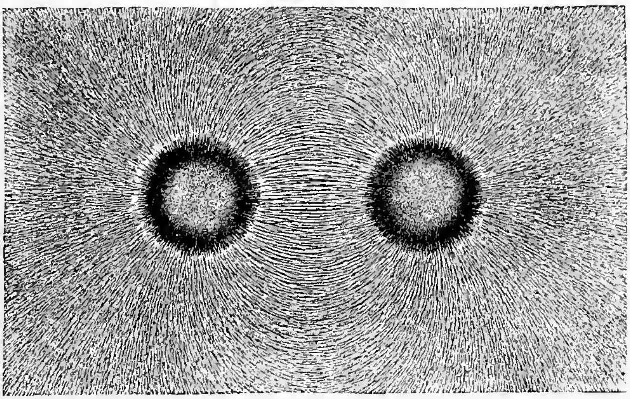

Fig. 159.

lines of force are as shown in Fig. 161, which shows a side view of two adjacent similar poles.

(*d*) In the neighborhood of a long, straight wire carrying an electric current, the lines of force are circles having their centers at the wire, and their planes perpendicular thereto, as shown in

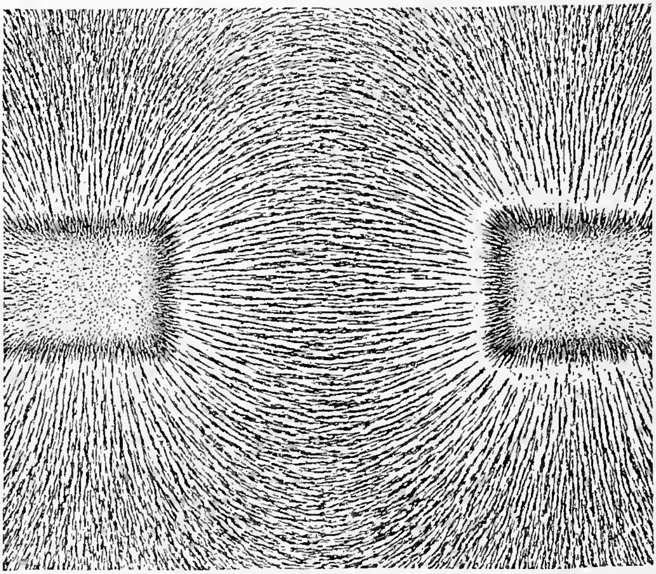

Fig. 160.

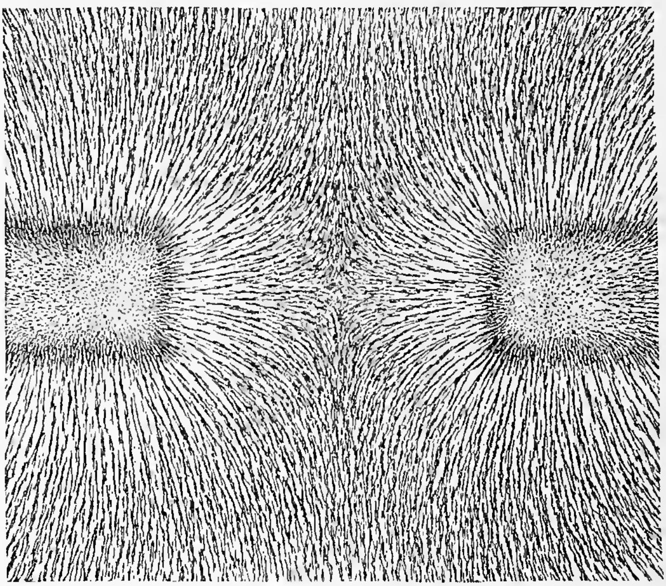

Fig. 161.

Fig. 162. The small circle at the center represents the wire, which is perpendicular to the plane of the figure. The immediate neighborhood of a short portion of any wire carrying an electric current is a magnetic field of this character.

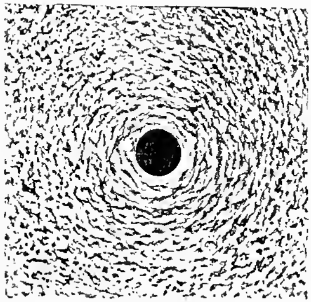

Fig. 162.

(*e*) Consider a long straight wire at right angles to a uniform magnetic field. If an electric current be sent through the wire, the field due to the current is superposed (see Art. 343) upon the uniform field, giving a magnetic field in which the lines of force trend as shown in Fig. 163. The undisturbed uniform field has the

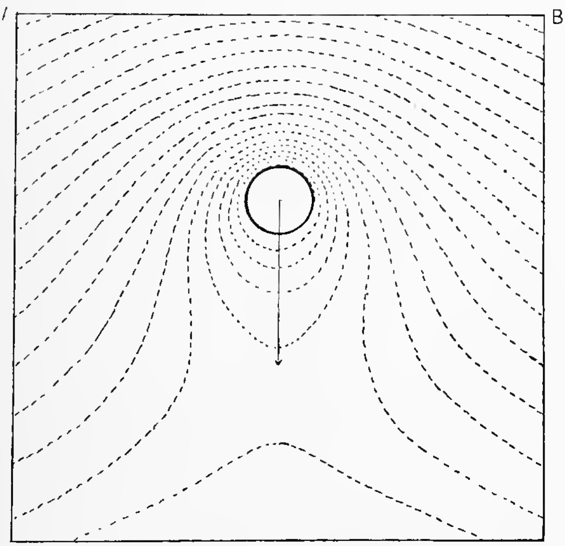

Fig. 163.

direction *AB*, and the small circle represents the wire. The latter is perpendicular to the plane of the figure.

(*f*) In the neighborhood of a cylindrical coil of wire carrying

a current, the lines of force pass through the opening of the coil and return on the outside.

350. Preliminary statement concerning the physical nature of magnetic fields and of the electric current. — There can be no doubt but that what we call magnetic field results from or is some kind of disturbance in the *ether.** Indeed, it is likely that *the ether, at each point in a magnetic field, rotates* with great angular velocity *about an axis* which is *parallel to the direction of the field at the point.* This assumption, together with others to follow, leads to a comprehensive theory of electricity and magnetism, and of light. Such a rotatory motion of the ether would tend to shorten the rotating portions in the direction of the axis and to expand them in all directions at right angles thereto. As a matter of fact, all magnetic phenomena indicate a tendency for the lines of force in a magnetic field to shorten, and a tendency for them to spread apart laterally. Study in this connection Figs. 158 to 161, remembering that unlike poles attract and that like poles repel one another.

351. The magnetic field as a seat of kinetic energy. — If the magnetic field is a whirling motion of the ether, it must be a seat of kinetic energy. Indeed, it can be proven † that

$$W = \frac{1}{8\pi} f^2, \qquad (198\,a)$$

in which W is the *kinetic energy per unit volume* in a magnetic field, in the neighborhood of a point at which the intensity of field is f.

* Magnetic fields can exist in a region devoid of ordinary matter; that is, in a vacuum. It is therefore necessary, if one is to reach a mechanical conception of the magnetic field, to assume the existence of a medium, the *ether*, which permeates all space.

† See Art. 463.

CHAPTER III.

THE ELECTRIC CURRENT.

352. Preliminary statements. — The phenomenon called the electric current has its seat largely in the region surrounding the wire in which the current is said to "flow." What takes place in the wire itself follows directly, as we shall see, from the nature of magnetic field and the manner of distribution of the field about the wire.

In studying the electric current we shall have occasion to distinguish the following properties:

(*a*) *Magnetic effect.* — A wire through which an electric current flows* is surrounded by a magnetic field of the character indicated in Art. 349 (*d*). *If the wire passes through a uniform magnetic field and at right angles thereto, as explained in Art.* 349 (*e*), *then when current flows, the wire is pushed sidewise in the direction of the arrow F* (Fig. 163). The origin of this force, as is evident from that figure, may conveniently be ascribed to the tendency of the lines of force to shorten or contract.

(*b*) *Heating effect.* — A wire through which an electric current flows has heat generated in it.

(*c*) *Chemical effect.* — If the wire is cut and the ends are dipped into a solution of a chemical compound, the compound is decomposed.

If the current passes through the human body, certain effects are produced, depending mainly upon the excitation of the nerves. This physiological effect is no doubt due either to the *heating effect* or to the *chemical effect* of the current, and is not to be classed along with these as fundamental.

* This word and also the word *current* arose from a misconceived analogy between the electric current and a current of liquid in a tube. Both terms will be used in their accepted sense.

353. Ampere's law. *Strength of current.*—The force F with which a wire is pushed sidewise when at right angles to a uniform magnetic field of intensity f, as described in Art. 349 (c), *is proportional to the product of the intensity of the field into the length l of the wire.* That is,

$$F = Ilf, \tag{199}$$

in which I is a constant under the given conditions. This quantity I is called the *strength* of the current in the wire.

Dimensions of strength of current.—If we substitute dimensions of F and f in (199), we may solve for I, thus finding its dimensions in terms of length, mass, and time. The derivation of dimensions of the various electrical quantities is in every case a simple matter and will be left to the reader henceforth.

Ampere further observed that a wire which carries a current and which lies parallel to a magnetic field is not affected thereby. Therefore, if a length Δl of wire, in a field of intensity f, makes an angle θ with f, then $f \sin \theta$ is the component of f perpendicular to the wire, and

$$\Delta F = If \cdot \sin \theta \cdot \Delta l \tag{200}$$

is the force pushing the wire in a direction perpendicular both to Δl and to f.

354. C. G. S. unit of current. *The ampere.*—When F, in equation (199), is expressed in dynes, f in c. g. s. units, and l in centimeters, I is said to be expressed in *c. g. s. units.* A *c. g. s. unit current* is therefore a current of such strength, that, flowing through a wire at right angles to a magnetic field of unit intensity, each centimeter of the wire is pushed sidewise with a force of one dyne. The *ampere*, which is the practical unit of current, is defined as *one-tenth* of a c. g. s. unit current.

355. Direction of current.—It is evident from Fig. 161 that a north magnetic pole tends to move in one direction round a

wire carrying a current, while a south pole tends to move round in the opposite direction. It is customary to speak of the current as flowing in that direction along the wire in which a right-handed screw (coaxial with the wire) would move if turned in the direction in which a north pole tends to move round the wire. It is often convenient to know that in an electrolytic cell metals are deposited upon the electrode towards which the current, by convention, flows. (See Art. 391.)

Figure 164 shows the relative direction of magnetic field f, electric current C (directed away from the reader perpendicular to paper) and direction of side force F acting on current.

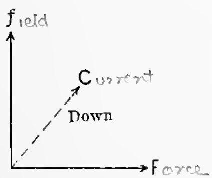

Fig. 164.

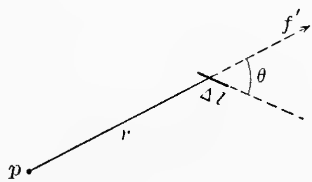

Fig. 165.

356. Contribution to magnetic field at a point by one element Δl of a wire carrying a current of strength I. — Let p (Fig. 165) be the point, distant r from the element Δl of wire, and let θ be the angle between r and Δl. Imagine a magnetic pole of strength m placed at p. The field f' at Δl due to this pole is $f' = \frac{m}{r^2}$ (from equation (193)). The element Δl in this field f' is acted upon by a force $\Delta F = If' \sin\theta \cdot \Delta l = I\frac{m}{r^2}\sin\theta \cdot \Delta l$ (from equation (200), and this force is perpendicular to the plane $r \cdot \Delta l$. Since this is the force with which the pole acts upon the element, the element must react upon the pole with an equal and opposite force. Therefore $\frac{Im \sin\theta \cdot \Delta l}{r^2}$ is the force acting upon the pole at p, due to the element of current. Since, moreover, the force acting upon a pole is equal to the product of the strength of the pole into the intensity of the field at the pole

due to the agent which is acting on the pole (see equation (192)), we have

$$\Delta f = \frac{I \sin\theta \cdot \Delta l}{r^2}, \tag{201}$$

where Δf is the field at p due to the element Δl.*

357. Magnetic field f at a point due to a wire carrying a current of strength I. — We have, from equation (201),

$$f = I \sum \frac{\sin\theta \cdot \Delta l}{r^2}. \tag{202}$$

This summation, being a vector summation, requires some further explanation. From the point p (Fig. 165), draw a *line* representing the value of $\Delta f = \dfrac{I \sin\theta \Delta l}{r^2}$ at that point due to a given element Δl of the current. The direction of Δf will be perpendicular to the plane $r \cdot \Delta l$. From the terminus of this line, draw another representing, in the same manner, the contribution to the field at p due to the next element of the current, and so on for the whole circuit. The line drawn from p to the point finally so reached will then represent the intensity of field at p due to the whole circuit.

Remark. — Since equation (202) contains I as a factor, it is clear that *the intensity of the magnetic field, at any given point in the neighborhood of a given coil of wire, is proportional to the*

* We have here considered the mutual action of a current element and a magnetic pole to be a force *at the element*, and an equal and opposite force *at the pole*. This apparently absurd result is due to the fact that the *current element* is to be considered strictly as a flow through Δl, and a free return throughout the surrounding region, as shown in Fig. 166. The force on this *complete* current element due to a pole acts *at the pole*, as does the equal and opposite force on the pole due to the element. The force with which the pole acts on the *incomplete* element Δl is at the element, and the force with which the pole acts upon the remainder of the *complete* element is a torque which just balances the *couple*, consisting of the force on the pole due to the element, and the force on the element due to the pole.

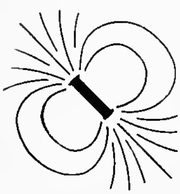

Fig. 166.

current I in the coil, and that the direction of the field does not vary with the strength of the current. It is often necessary to consider the intensity f of magnetic field, at the center of the coils of a galvanometer, due to a current I in the coils. From the above statement, we have

$$f = GI, \tag{203}$$

in which G is a constant for the given coils. This constant is called the *constant of the coils.*

358. Strength of field at the center of a circular coil carrying a current of strength I. — In this case, since r has the same value for each element, and θ is everywhere 90°, equation (202) becomes $f = \frac{I}{r^2}\sum \Delta l$. But $\sum \Delta l = 2\pi nr$, in which n is the number of turns of wire in the coil. Therefore the equation takes the form

$$f = \frac{2\pi nI}{r} \tag{204}$$

The field f is, of course, perpendicular to the plane of the coil.

It is instructive to derive equation (204) directly from (199) without explicit use of (201). This may be done as follows:

Imagine a magnetic pole of strength m placed at the center of the coil. The field *at the wire* due to this pole is $\frac{m}{r^2}$, and a length $2\pi nr$ of wire is in this field and everywhere perpendicular to it, so that the coil is pushed by the pole with a force $F = I\,2\pi nr \cdot \frac{m}{r^2}$ parallel to the axis of the coil (see (199)). An equal and opposite force acts on the pole due to the coil, and the force which acts on the pole is equal to fm (from equation (192)); therefore $f = \frac{2\pi nI}{r}$. The field f, at a point in the axis of a circular coil, of radius r, at a distance d from the plane of the coil, is easily shown to be

$$f = \frac{2\pi nr^2 I}{(r^2 + d^2)^{\frac{3}{2}}}. \tag{205}$$

359. Galvanometers. — The field within a coil carrying current affords, by its action upon a magnet needle suspended at the center, or in the axis of the coil, a convenient means of measuring the current. Instruments for thus measuring current are termed galvanometers. An important type of galvanometer, which depends for its action directly upon equation (204), is the tangent galvanometer.

360. The tangent galvanometer. — This instrument consists essentially of a circular coil of wire, sometimes of a single turn.

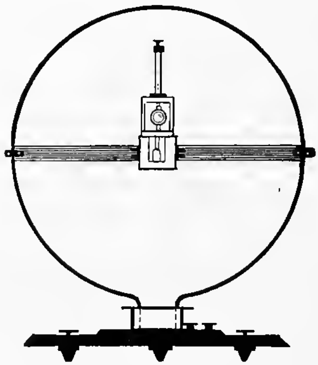

Fig. 167.

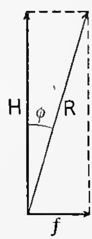

Fig. 168.

This is mounted so that its plane is vertical and in the direction of the horizontal component H of the earth's magnetic field. Figure 167 shows the arrangement of a simple form of tangent galvanometer. A small magnet suspended at the center of the coil points in the direction of H. A current I being sent through the coil of a field $f = \frac{2\pi n I}{r}$ at right angles to H is produced thereby. This field compounded with H gives a resultant field R (Fig. 168), in the direction of which the small magnet now points, it having been turned through the angle ϕ by the current so that $\tan\phi = \frac{f}{H}$. Writing the value $\frac{2\pi n I}{r}$ for f in this equation and solving for I, we have

$$I = \frac{rH}{2\pi n}\tan\phi. \tag{206}$$

This equation gives I when r, H, and n are known, and ϕ is observed. If I is to be expressed in amperes, then

$$I_{\text{amp}} = \frac{5\,rH}{\pi n} \tan\phi\,; \tag{206a}$$

for the number which expresses a current in amperes must be ten times as large as the number which expresses the same current in c. g. s. units.

361. The Helmholtz form of tangent galvanometer. — To secure uniformity of field, tangent galvanometers are frequently constructed with two parallel coils. These are mounted vertically and in the magnetic meridian. Their distance apart ($2\,x$, Fig. 169) is equal to their radius r. The needle is suspended midway between them in their common axis. In the figure, looking down upon the instrument, the coils ab and cd are shown in cross-section. They are at right angles to the paper. The needle is at NS. This form of galvanometer is due to Helmholtz* and to Gaugain.†

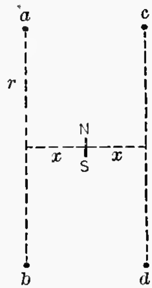

Fig. 169.

The character of the fields produced by a single coil and by two coils placed as above is shown in Figs. 170 and 171. When two coils are used in a galvanometer, the deflecting field f (equation (205)) is the sum of those which would be produced by each coil separately. Equation (205) takes the form

$$I = \frac{H}{\dfrac{2\,\pi r_1^2 n_1}{(r_1^2 + d_1^2)^{\frac{3}{2}}} + \dfrac{2\,\pi r_2^2 n_2}{(r_2^2 + d_2^2)^{\frac{3}{2}}}} \tan\phi, \tag{207}$$

in which r_1, r_2 are the radii of the coils, n_1, n_2 are the number of turns of wire in each, and d_1, d_2 are the perpendicular dis-

* Helmholtz, Sitsungsberichte der Physikalischen Gesellschaft in Berlin, 1849.
† Gaugain, Comptes Rendus, 36, p. 191, 1853.

tances of the needle from the planes of the respective coils. However many coils may be employed, and whatever the number of turns in each may be, and whatever their radii and distances from the needle, the current may always be computed

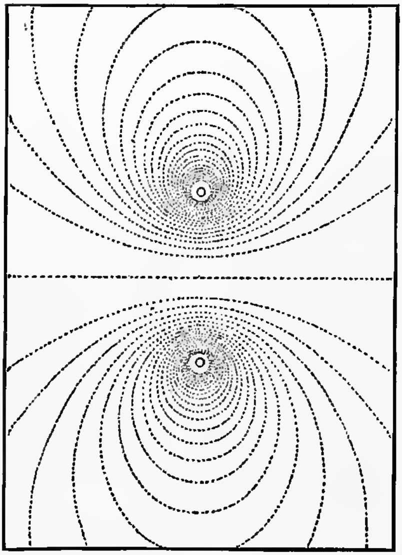

Fig. 170.

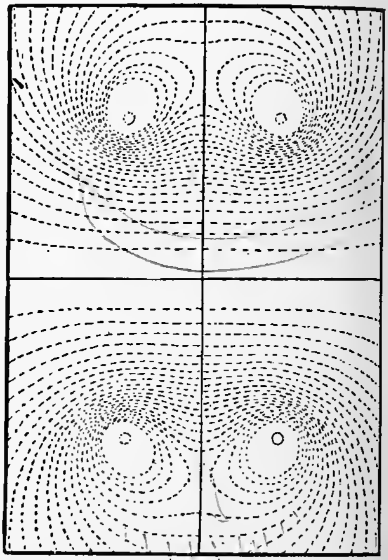

Fig. 171.

by means of an equation of the form of (207), provided the coils are all parallel and in the magnetic meridian and they have a common axis, in which the needle is suspended. The more general form of (207) is

$$I = \frac{H}{\frac{2\,\pi r_1^2 n_1}{(r_1^2 + d_1^2)^{\frac{3}{2}}} + \frac{2\,\pi r_2^2 n_2^2}{(r_2^2 + d_2^2)^{\frac{3}{2}}} + \frac{2\,\pi r_3^2 n_3^2}{(r_3^2 + d_3^2)^{\frac{3}{2}}} + \text{etc.}} \tan\phi. \quad (207\,a)$$

The denominator in the right-hand member of this equation is sometimes called the **constant of the coils.** It is designated by the letter G. The quantity $\frac{H}{G} = K$, by which the tangent

of the deflection is multiplied to find the value of I, is called the **constant of the galvanometer**, or sometimes the **reduction factor.**

The latter quantity depends upon the value of H, and therefore upon the location of the galvanometer and the time. The constant G, however, depends only upon the dimensions of the instrument; it is independent of time and place.

362. The comparison of two currents by means of the tangent galvanometer. — The use of the tangent galvanometer for measuring currents, absolutely, has been explained in Art. 360. It may also be used for the comparison of two currents. Equation (207) may be written

$$I = K \tan \phi, \tag{208}$$

in which $K\left(\text{written for } \frac{rH}{2\pi n}\right)$ is the reduction factor. If another current I' be sent through the galvanometer and the corresponding deflection ϕ' be observed, we have $I' = K \tan \phi'$. Dividing equation (208) by this equation, member by member, we obtain

$$\frac{I}{I'} = \frac{\tan \phi}{\tan \phi'}, \tag{209}$$

by the use of which the tangent galvanometer may be used to measure the *ratio* of two currents, even when the reduction factor is not known.

The reduction factor K of a tangent galvanometer may be determined experimentally by observing the deflection ϕ produced by a current which is measured by some other method. The value of K is then given by equation (210).

363. Behavior of a circular coil carrying current, when suspended in a uniform field. — Consider a circular coil, of radius r (Fig. 172), which is mounted upon a vertical axis AB, with its plane in the direction of a uniform magnetic field H. The

action of this field on the coil is a torque about AB, the value of which is,

$$T = \pi n r^2 I H, \tag{210}$$

where I is the current in the coil and n the number of turns of wire.

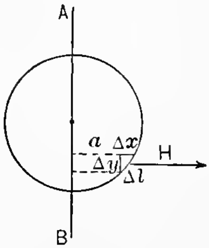

Fig. 172.

Proof. — Consider an element Δl of one turn of wire. Resolve this element into vertical and horizontal components, Δx and Δy (this is allowable if the element is very short). The field has no action on Δx, which is parallel to H (see Art. 353). The force acting on Δy is $IH \cdot \Delta y$ (from equation (199)). The torque action of this force about AB is $IHa \cdot \Delta y$, and the total torque action on one turn of wire is $\sum IHa \cdot \Delta y$ or $IH \sum a \cdot \Delta y$. But $\sum a \cdot \Delta y$ is the area πr^2 of the circle formed by the coil, so that $\pi r^2 IH$ is the torque acting on each turn, and $\pi n r^2 IH$ is the total torque acting on the coil.

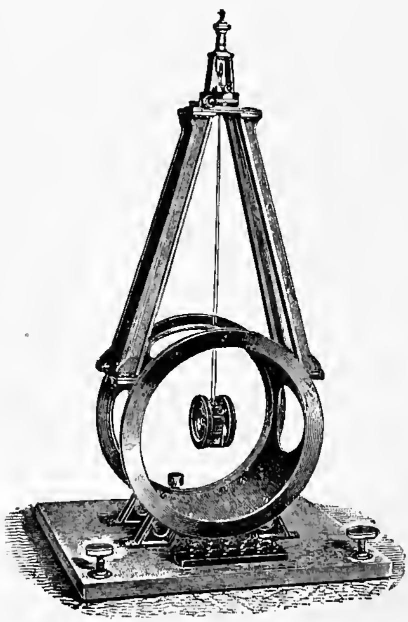
Fig. 173.

364. The electrodynamometer. — The behavior just described of a coil carrying current, in a magnetic field, makes it possible to construct an instrument, analogous to the galvanometer, in which the moving part is a coil of wire. Wilhelm Weber, of Goettingen, devised such an apparatus (1846), which is known as

Weber's electrodynamometer. It consists of a large circular coil, rigidly mounted, with its plane vertical, and a small circular coil suspended at its center, the planes of the two coils being at right angles. Figure 173 shows a later and slightly modified form. A current I is sent through both coils. The field produced by the outer coil, at its center, is $f = \frac{2\pi n' I}{r'}$ (see equation (204)), where n' is the number of turns in the coil, and r' is its radius. This field exerts a torque $T = \pi n'' r''^2 I f$ upon the small coil (see equation (210)), where n'' is the number of turns in the small coil, and r'' is its radius.

If we substitute the value $f = \frac{2\pi n' I}{r'}$ in this expression for T, we have

$$T = \frac{2\pi^2 n' n'' r''^2 I^2}{r'}. \tag{211}$$

This equation permits the calculation of I when n', n'', r', and r'' are known, and T has been observed.

365. Sensitive galvanometers. — In the instruments already described (Arts. 316 *et seq.*) the construction is such as to permit of the computation of the intensity of a current from the dimensions of the galvanometer, a knowledge of the value of H, and the observation of the deflection. Where very small currents are to be measured it becomes necessary to modify the construction.

By inspecting the equation (206) of the tangent galvanometer it is evident that a given current will produce the greatest deflection ϕ when the number of turns of wire in the coil is very great, when the radius of the coil is very small, and when the directing field H is weak. A galvanometer constructed in this way is called a sensitive galvanometer.

The magnet of such an instrument is suspended by means of a fine fiber of unspun silk or of quartz. In order that small deflections may be easily detected it is customary to attach a small mirror to the suspended magnet, and to observe with a

telescope and scale (see Vol. I. p. 5). Sometimes the magnet is made in the form of a steel disk (Fig. 174), one face of which is polished and serves as a mirror. Sometimes a mirror of thin glass is employed, and the magnet or a set of small parallel magnets is cemented to its back (Fig. 175). Sometimes mirror and magnets are attached to a light vertical rod (Fig. 176). The last named plan has the advantage of removing the mirror from the center of the coil and making it possible to bring the latter nearer to the magnet.

Use of governing magnets. — In order to secure a weak directing field H, the earth's field may be partially neutralized in the neighborhood of the suspended magnet needle by superposing upon it an opposing field due to a large magnet rightly placed

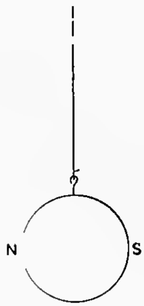

Fig. 174.

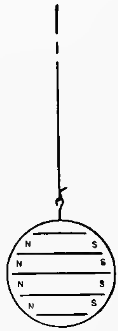

Fig. 175.

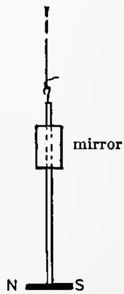

Fig. 176.

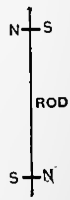

Fig. 177.

in the neighborhood of the galvanometer. This magnet is called a *governing* magnet. This device, however, introduces certain difficulties. The earth's field is slightly variable, while the opposing field, due to a governing magnet, is constant. The fluctuations in H become very troublesome, therefore, when the attempt is made to gain extreme sensitiveness by the use of a governing magnet.

Use of astatic systems of magnets. — A method for further increasing the sensitiveness of galvanometers, consists in the use of what is termed an astatic system of magnets.

Two magnets, NS and SN, of equal magnetic moments, at-

tached to a rod, as shown in Fig. 177, constitute an astatic system. Such a system if suspended in the earth's field will point indifferently in any direction. If one magnet is slightly stronger than the other, or if their axes do not lie in the same plane, the earth's field will exert only a very slight directing action upon the system. Such a system may be suspended with one of its magnets inside of a galvanometer coil as shown in Fig. 178; or two coils, so connected that a current sent through the instrument flows in opposite directions in them, may be used, one surrounding each magnet, as shown in Fig. 179. This latter design is due to Sir W. Thomson (Lord Kelvin). A galvanometer so constructed, with very short magnets, light connecting rod and mirror, and coils containing many turns of wire, can be made to indicate distinctly currents which do not exceed 10^{-12} amperes. To attain such a degree of sensitiveness, however, the coils must be brought very closely together, so that there will be barely room enough for the magnets to hang freely between them.

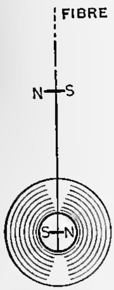

Fig. 178.

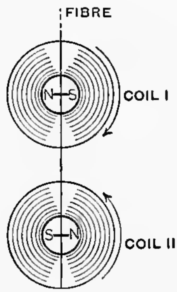

Fig. 179.

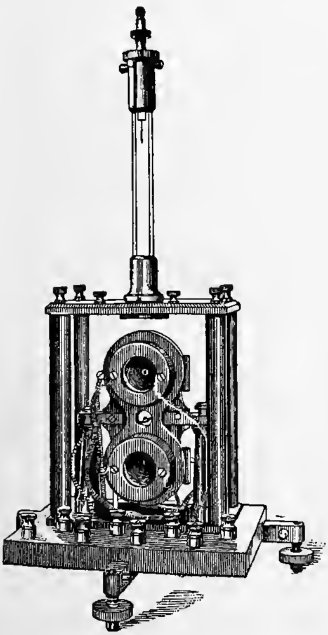
Fig. 180.

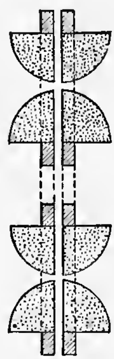
Fig. 181.

Figures 180 and 181 show the arrangement of a galvanometer designed with a view to the fulfillment of the conditions of extreme sensitiveness.

It has an astatic system of magnets, with an exceeding small and light mirror upon the vertical rod, midway between them (Fig. 182). Each group of magnets lies at the center of a pair of coils of wire, the clearance space not exceeding 1 mm. The suspended parts, which weigh, taken altogether, but a few milligrams, hang from a fiber of quartz so fine as to be invisible to the naked eye.*

Fig. 182.

Such instruments do not follow strictly the law of the tangent galvanometers. Since, however, for small range, the deflection is always sensibly proportional to the current, they may be employed for the approximate measurement, as well as the detection, of weak currents.

366. The D'Arsonval galvanometer. — In order to avoid the difficulties incident to the use of a weak directing field, galvanometers are frequently constructed in accordance with the following principle. They are known as *D'Arsonval galvanometers.*

It has already been shown that a coil suspended on a vertical axis in the earth's field is acted upon by a torque,

$$T = \pi n r^2 IH. \tag{210 bis}$$

If the coil is suspended by wires, this torque will produce a slight movement of the coil. In order that the coil may be perceptibly moved by a very weak current, the suspending wires, which also serve to lead current to and from the coil, must be very fine, the number of turns of wire in the coil must be great, and the field H in which the coil is placed must be very strong. To obtain a quick movement of the coil it is important to have its lateral dimensions small. In accordance with these conditions Ayrton and Sumpner have recommended the following design.

* For further details see Nichols, The Galvanometer, Lecture 6.

An elongated coil with attached mirror is suspended in the strong field between the poles of a horseshoe magnet as shown in Fig. 183. This galvanometer is by no means so sensitive as that described in Art. 365, but it is scarcely at all affected by outside magnetic influences. It may be used indeed in a room with any kind of electrical machinery without inconvenience, if it be rigidly mounted. This galvanometer may be used for

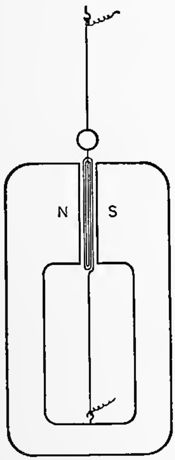

Fig. 183.

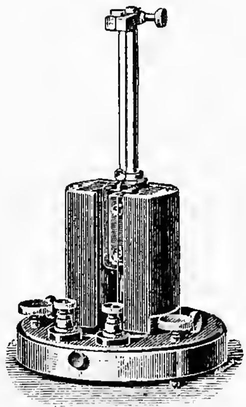

Fig. 184.

the approximate measurement of weak currents. The deflection within a small range is sensibly proportional to the current.

Figure 184 shows a modern galvanometer of the D'Arsonval type. The coil is very long and narrow. It is suspended in field produced by several similar horseshoe magnets placed one above another. The deflections are read by means of a telescope and scale.

CHAPTER IV.

RESISTANCE AND ELECTROMOTIVE FORCE.

RESISTANCE.

367. Joule's law; definition of resistance. — The rate at which heat is generated in a given wire is proportional to the square of the current flowing in the wire; that is,

$$\dot{H} = RI^2, \tag{212}$$

in which $\dot{H}$ is the rate of generation of heat in a wire by a current of strength I, and R is the proportionality factor. This quantity R is called the *resistance* of the wire.

If the current I is constant, then the amount of heat H generated in the wire during t seconds is $H = \dot{H}t = I^2Rt$; that is,

$$H = RI^2t. \tag{213}$$

The name *resistance* had its origin in the antiquated conception, according to which the electric current consists in a *flow* of some imaginary or hypothetical fluid. In passing through a circuit this fluid was supposed to suffer obstruction in a manner analogous to the frictional obstruction to the flow of water through pipes.

368. C. G. S. unit of resistance; the ohm. — If we express $\dot{H}$ in equation (212) in *ergs per second*, and I in c. g. s. units of current, R is said to be expressed in *c. g. s. units of resistance*. Therefore a wire has one c. g. s. unit of resistance when heat is generated in it at the rate of one erg per second by one c. g. s. unit of current. The *ohm*, which is the unit chiefly used in practice, is 10^9 c. g. s. units of resistance. Therefore if I in equation (212) is expressed in *amperes*, and R in *ohms*, $\dot{H}$

will be expressed in terms of a unit, which is equal to 10^7 ergs per second; that is, in *watts*. See Vol. I. p. 59, for definition of the watt.

369. Specific resistance. — The resistance R of a wire of a given material is found to be directly proportional to its length l, and inversely proportional to its sectional area q; that is,

$$R = K\frac{l}{q}. \tag{214}$$

The proportionality factor K is called the *specific resistance* of the material. It is equal to the resistance of a wire of unit length ($l = 1$) and unit sectional area ($q = 1$).

The accompanying table gives the value of K (resistance in ohms of a wire one centimeter long and one square centimeter sectional area) for various substances.

Remark. — The reciprocal of the resistance of a circuit is called its *conductivity.* The conductivity of a wire is directly proportional to its sectional area and inversely proportional to its length. The proportionality factor is called the *specific conductivity* of the material. Specific conductivity is the reciprocal of specific resistance.

TABLE OF SPECIFIC RESISTANCE (RESISTIVITY).

METAL.	RESISTANCE OF A BAR 1 CM. LONG, 1 SQ. CM. CROSS-SECTION, AT 0° C.
Aluminium (annealed)	0.00000289 ohm.
Copper (annealed)	0.00000160
Gold	0.00000208
Iron (pure)	0.00000964
Iron (telegraph wire)	0.00001500
Lead	0.00001963
Mercury	0.00009434
Platinum	0.00000898
Silver (annealed)	0.00000149
German Silver (Cu 60, Zn 26, Ni 14)	0.000021 ohm.
Platinoid (Cu 59, Zn 25.5, Ni 14, W 55)	0.000032
Manganin (Cu 84, Ni 12, Mn 3.5)	0.000047

370. Influence of temperature upon resistance. — The resistance of a circuit depends not only upon the material of which the circuit is made, but also upon the temperature of the mate-

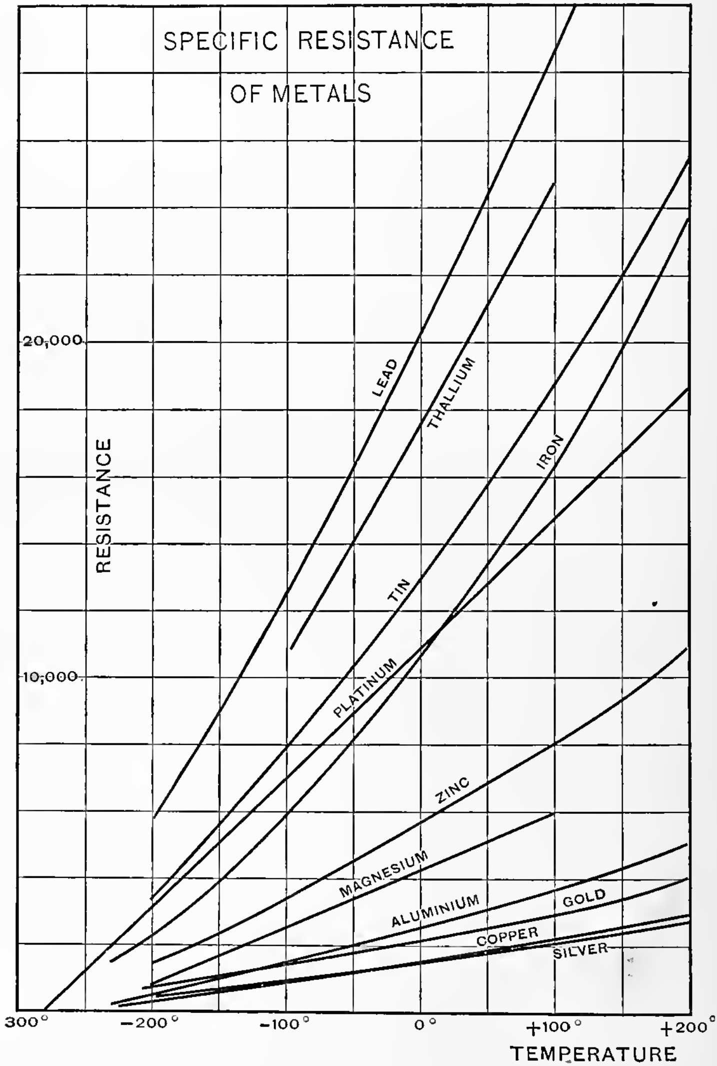

Fig. 185

rial. The increase in the resistance of a given circuit, due to a rise in temperature, is proportional to the initial resistance and approximately proportional to the rise in temperature; that is, if R_0 is the resistance of a circuit at some standard temperature, say at zero centigrade, then the increase of resistance when the circuit is warmed to $t°$ C. is $\beta R_0 t$, where β is the proportionality factor. Therefore the total resistance of the circuit at $t°$ C. is $R_t = R_0 + \beta R_0 t$, or,

$$R_t = R_0 (1 + \beta t). \tag{215}$$

The quantity β is called the *temperature coefficient of resistance of the given material.* For all pure metals, except iron, β has nearly the same value, viz., .00366; that is, the resistance of a pure metal is very nearly proportional to the absolute temperature, as measured by an air thermometer. For iron, and also many specimens of copper, β has a value which is somewhat greater than this.

Electrolytes and graphitic carbon diminish in resistance with rise of temperature, so that these substances have negative temperature coefficients of resistance. (See Art. 371.)

The influence of temperature upon resistance has been thoroughly investigated between $-200°$ and $+200°$. Figure 185 gives a graphic representation of the results obtained by Dewar and Fleming.* The ordinates of the curves are specific resistances. They are expressed in c. g. s. units, and to be reduced to ohms must therefore be divided by 10^9.

371. Specific resistances and temperature coefficients of alloys. — Alloys of lead, tin, cadmium, and zinc act as though they were simple mixtures. For example, the straight line (Fig. 186)† marked Tin-Zinc indicates that a mixture of these metals possesses a resistance intermediate to those of tin and zinc, and proportional to the percentage of mixture. Alloys of most

* Dewar and Fleming, Philosophical Magazine (5), Vol. 34, p. 326; Vol. 36, p. 271. Also Price, Measurement of Electrical Resistance, p. 17.

† Representing the results of Matthiesen, Philosophical Transactions, 1862.

other metals act very differently. The addition of one metal, even in small quantities, to any other increases the specific resistance and diminishes the temperature coefficient. The case of gold and silver may be taken as a type. It will be seen from Fig. 186 that the alloying of either pure gold or pure

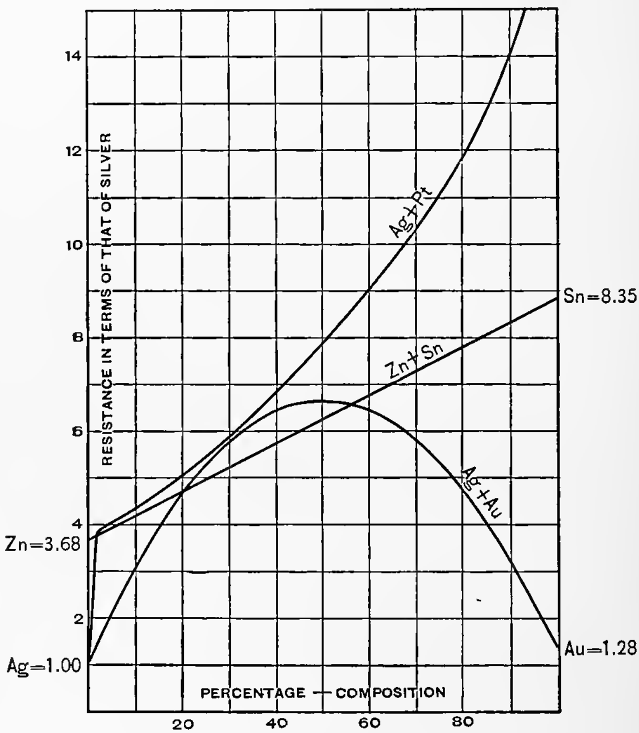

Fig. 186.

silver with a small percentage of the other metal causes a great increase in resistance.

By the use of manganese, alloys may be produced, the temperature coefficients of which are very small. The coefficient may be given a positive or negative value at will by slightly varying the composition of the alloy or its temper.

372. Expression, in terms of current and resistance, of the power expended in maintaining a current in a circuit. — It sometimes happens that all the work expended in maintaining a current in a circuit goes to heat the circuit. In such cases the power P so expended is equal to the rate, RI^2, at which heat is generated in the circuit, so that

$$P = RI^2. \tag{216}$$

When I and R are expressed in c. g. s. units, P is given in ergs per second. When I is expressed in amperes and R in ohms, P is given in watts.

373. Measurement of resistance. — The fundamental method for determining the resistance of a wire is by calculation from equation (213); viz., $H = RI^2t$, H, I, and t having been determined by observation. For example, the wire of which the resistance is to be determined may be submerged in a calorimeter, by means of which the heat H generated by a measured current I during t seconds may be determined. A calorimeter used in this way is called an *electro-calorimeter*. In its usual form this instrument consists of a cylindrical vessel C, which contains the coil of wire w the resistance of which is to be measured and which is nearly filled with some liquid, preferably an oil. A stirrer s is used to bring the temperatures of the interior to uniformity. The thermometer t indicates the rise in temperature. (See Fig. 187.)

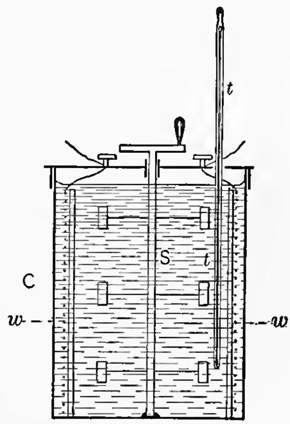

Fig. 187.

The electro-calorimeter may also be used for the measurement of current, in which case the resistance must be known (see

Art. 414), and for the determination of the mechanical equivalent of heat.*

Indirect methods for measuring resistance have been devised by Weber, Lorenz, and others.† Standard resistances have been determined with great care by these methods, and all ordinary laboratory methods for determining resistance are by comparing the resistance to be determined with a standard.

Electromotive Force.

374. Ohm's law; definition of electromotive force. — The current produced in a circuit by a battery, or by any agent, is inversely proportional to the resistance of the circuit. That is,

$$I = \frac{E}{R}, \tag{217}$$

or

$$RI = E, \tag{218}$$

in which E is a constant for the given battery, or agent. This quantity E is called the *electromotive force* of the battery, or agent.

An independent conception of electromotive force will be developed in Chapter XII. In the experimental work which led Ohm to the formulation of his law, he made use of this independent conception of electromotive force, and his whole discovery was that *the current produced by an agent is directly proportional to the electromotive force of the agent, and inversely proportional to the resistance of the circuit.*

In subsequent articles, the convenient abbreviation e.m.f. will be used for electromotive force.

375. Ohm's law as applied to a portion of a circuit. — Consider a portion AB (Fig. 188) of a circuit through which a

* See Nichols, Laboratory Manual, Vol. II. p. 270.

† See A. Gray, Absolute Measurements in Electricity and Magnetism, Vol. II. pp. 538–600.

current I is flowing. Let R be the resistance of this portion. Then, in accordance with equation (218), the product RI is called the electromotive force between the points A and B. For the present, the student will find it helpful to consider the e. m. f. between two points of a circuit as analogous to the *difference in pressure between two points* of a pipe through which liquid is flowing.

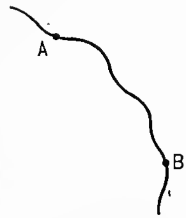

Fig. 188.

376. Units of e. m. f. — If I, in equation (218), is expressed in c. g. s. units of current, and R in c. g. s. units of resistance, E is said to be expressed in *c. g. s. units of e. m. f.* Therefore, the c. g. s. unit of e. m. f. is the e. m. f. between the ends of a wire having 1 c. g. s. unit of resistance, and through which 1 c. g. s. unit of current is flowing.

The volt. — If i, in equation (218), is expressed in amperes, and R in ohms, E is said to be expressed in **volts.** Therefore, *the volt is the e. m. f. between the ends of a wire having one ohm resistance, and through which one ampere of current is flowing.* The volt is equal to 10^8 c. g. s. units of e. m. f. It is the unit chiefly employed in practical work in electricity.

377. The measurement of e. m. f. — Electromotive force is determined by measuring the current i, which must be sent through a wire of known resistance R, in order that the e. m. f., Ri, between the ends of the wire, may be equal to the e. m. f. to be determined. The e. m. f. to be determined is then given by equation (218). For detailed directions for carrying out this method, see Art. 423.

378. Expression, in terms of E and I, of the power P expended in maintaining a current in a circuit. — If we substitute the value of R from equation (218), viz., $R = \frac{E}{I}$, in equation (216), we have,

$P = RI^2$

$$P = EI, \tag{219}$$

in which P is the power expended in maintaining a current I in any portion of a circuit, and E is the e. m. f. between the ends of that portion. If E and i are in c. g. s. units, P will be expressed in ergs per second. If E is in volts and I in amperes, P will be expressed in watts.

The derivation here given of equation (219) assumes all the power to be spent in heating the circuit, *i.e.* in overcoming resistance. In the course of the chapters on electrostatics, however, we shall come to recognize e. m. f. primarily through the relation (219), and this equation will then be found to be entirely general.

Shunts.

379. Kirchhoff's law. — Consider a point p (Fig. 189) at which a circuit A branches. *The strength of current in A is equal to the sum of the strengths of current in the various branches.* If current be reckoned positive when flowing toward a branch point p, and negative when flowing away from it, then the sum of the strengths of the currents in wires meeting at a branch point is equal to zero.

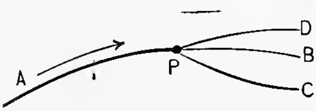

Fig. 189.

Another statement of Kirchhoff's law is that the strength of a current is the same in all parts of a circuit, branches, if they exist, being reckoned together. An important corollary to Kirchhoff's law, the truth of which has been tacitly assumed in previous articles, is as follows:

Established currents flow only in closed or completed circuits. See Art. 388.

380. Problem. — To determine the currents in each of two branches of a circuit in terms of the total current and of the resistances of the respective branches. Consider a circuit carrying a current i, which branches at the points A and B,

as shown in Fig. 190. Let i' be the current in the upper branch, and r' its resistance. Let i'' be the current in the

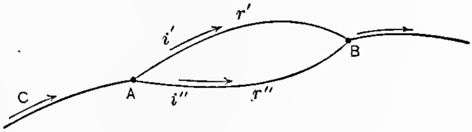

Fig. 190.

lower branch, and r'' its resistance. *Let E be the e.m.f. between the branch points A and B.* Then, from equation (218), we have,

$$E = i'r', \tag{i}$$

and

$$E = i''r''. \tag{ii}$$

From Kirchhoff's law we have also,

$$i = i' + i''. \tag{iii}$$

Eliminating E from (i) and (ii), we may use the resulting equation, viz., $i'r' = i''r''$, together with (iii), and determine the values of i' and i'' in terms of i, r', and r''. We thus obtain

$$\left. \begin{aligned} i' &= \frac{r''i}{r' + r''}, \\ i'' &= \frac{r'i}{r' + r''}. \end{aligned} \right\} \tag{220}$$

381. Combined resistance of a number of branches of a circuit defined. — Consider a circuit having, say, three branches be-

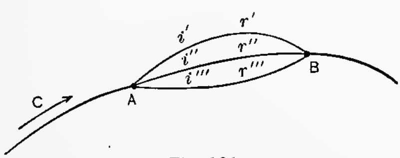

Fig. 191.

tween the points A and B. Let E be the e. m. f. between A and B. Then from equation (218) we have,

$$i' = \frac{e}{r'}, \tag{i}$$

$$i'' = \frac{e}{r''}, \tag{ii}$$

$$i''' = \frac{e}{r'''}. \tag{iii}$$

The combined resistance R of these branches is defined as a resistance which would allow the whole current i to flow from A to B with the same e.m.f. E between A and B. That is,

$$i = \frac{E}{R}. \tag{iv}$$

From Kirchhoff's law, $i = i' + i'' + i'''$, so that

$$\frac{E}{R} = \frac{E}{r'} + \frac{E}{r''} + \frac{E}{r'''}, \tag{v}$$

or

$$R = \frac{1}{\frac{1}{r'} + \frac{1}{r''} + \frac{1}{r'''}}. \tag{221}$$

382. Use of shunts with galvanometers. — If a galvanometer for detecting current is too sensitive, or if the current which must be used is greater than a galvanometer will indicate, the remedy is to *shunt* the galvanometer. For this purpose, a portion of the total current coming in on A (Fig. 192) is allowed to pass through a branch S, called a shunt, and out on B. The remainder of the current passes through the galvanometer.

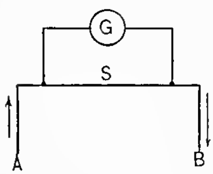

Fig. 192.

Let S be the resistance of the shunt, g the resistance of the galvanometer, i_1 the current flowing through the galvanometer, and i the total current. Then from equations (220) we have

$$i_1 = \frac{S}{S+g} i. \tag{222}$$

CHAPTER V.

THE ELECTRIC CHARGE; ELECTROLYSIS; BATTERIES.

383. Electric charge. — An electric current is conceived to be a transfer of electricity or of *electric charge* along a wire. The amount of charge ΔQ which in time Δt flows past a point in a wire carrying a current of strength I is, by definition,* $\Delta Q = I \cdot \Delta t$, whence $\frac{\Delta Q}{\Delta t} = I$, or

$$\frac{dQ}{dt} = I. \qquad (223)$$

That is, the *current in a wire is equal to the rate* $\left(\frac{dQ}{dt}\right)$ *at which electric charge is passing a given point in the wire.* The total charge, Q, which passes a point in a wire during a given interval of time is $Q = \sum I \cdot \Delta t$, or, if the current is constant,

$$Q = I \cdot t. \qquad (224)$$

384. C. G. S. unit of charge. The coulomb. — I in equation (224) being expressed in c. g. s. units of current, and t in seconds, Q is said to be expressed in *c. g. s. units of charge.* If I is expressed in amperes, and t in seconds, Q is expressed in terms of a unit called the *coulomb.* The coulomb is therefore the charge which flows in one second through a wire carrying one ampere of current. In specifying the discharge capacity of storage batteries the *ampere-hour* is a more convenient unit than the coulomb.

* An independent conception of electric charge will be developed in Chapter VII. The definition here given is found by experiment to be consistent with this independent conception.

385. Measurements of charge. — In the determination of very large charges, for example in the determination of the discharge capacity of a storage battery, the time t may be observed during which the given charge will maintain a sensibly constant measured current. The value of Q is then given by equation (224).

The charges which are oftenest encountered in practice are too small to be measured in this way. For their measurement the *ballistic galvanometer* is employed.

386. The ballistic galvanometer is a galvanometer arranged for measuring the total charge transferred during the flow of an electric current of short duration. The term should be applied to the method rather than to the instrument, since any galvanometer may be used ballistically. The method, however, demands certain qualities which are not essential nor desirable in galvanometers intended for use in the method of permanent deflections. Thus there has arisen a type of galvanometer especially designed for the ballistic method.

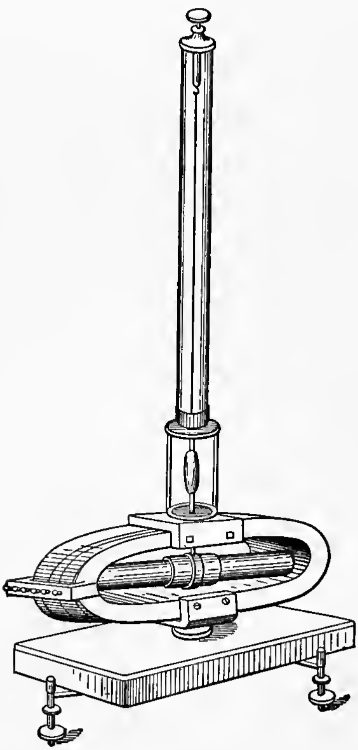

Fig. 193.

Figure 193 shows a type of galvanometer frequently used in ballistic work. The needle is a long bar magnet, such as is used in magnetometers. It is suspended between flattened coils by means of a stout fiber of silk. A mirror, the movement of which is observed with a telescope and scale, is attached to the magnet. The following discussion applies to a

galvanometer with a small magnet at the center of the coils.

The coils of the galvanometer are mounted with their plane vertical and in the direction of a horizontal magnetic field of intensity H; for example, the earth's field or a field produced by governing magnets. A current i in the coils produces a magnetic field at the center of the coils which is at right angles to H, and of which the intensity is

$$f = Gi \tag{i}$$

where G is the constant of the coils. (See Art. 357.) Let M be the magnetic moment of the galvanometer magnet. The field f, perpendicular to the axis of the needle, exerts upon it a torque equal to fM or to GiM by equation (195); and this torque, being unbalanced, is equal to the product of the moment of inertia K of the magnet into its angular acceleration. Therefore, writing $\frac{dQ}{dt}$ for i, we have

$$K\frac{d\omega}{dt} = GM\frac{dQ}{dt}, \tag{ii}$$

in which ω is the angular velocity, and $\frac{d\omega}{dt}$ the angular acceleration of the suspended magnet. If ω is zero at the instant of closing the galvanometer circuit, equation (ii) gives*

$$K\omega = GMQ, \tag{iii}$$

which is the fundamental equation of the ballistic galvanometer. If G, M, and K were known, Q could be calculated from an observed value of ω.

* This simple case of integration will occur often in subsequent chapters. The nature of the operation, plainly stated, is as follows: Equation (ii), if we divide both terms by GM, gives $\frac{K}{GM}\frac{d\omega}{dt} = \frac{dQ}{dt}$. It means in this form that ω *increases* $\frac{K}{GM}$ *times as fast as* Q. Therefore, *if* ω *and* Q *start from zero together,* ω *must always be* $\frac{K}{GM}$ *times as large as* Q, which is equation (iii).

In order to render the ballistic method feasible it is necessary to avoid the determination of the moment of inertia K and the magnetic moment M of the needle, and also the impracticable operation of observing the angular velocity ω of the needle. This is accomplished as follows:

(A) From equation (197) we have

$$\frac{4\,\pi^2 K}{\tau^2} = MH. \tag{iv}$$

(B) A constant known current I sent through the ballistic galvanometer will produce a permanent deflection ϕ of the needle, such that

$$\tan\phi = \frac{GI}{H}; \tag{v}$$

for the current I produces a field of intensity GI at right angles to H, and the needle turns into the direction of the resultant field.

(C) The kinetic energy of the moving needle after it has attained angular velocity ω is $\frac{1}{2}K\omega^2$. (From equation (64), Vol. I.) If there is no damping of its motion, the needle will turn until all this energy is spent in pulling it out of the direction of the field H. To move the north pole N (Fig. 194), the strength of which is m, from a_1 to a_2, and the south pole S of like strength, over a similar path, requires work against the force Hm. This work amounts to

$$2\,Hm \times a_1a_2 = HM(1 - \cos\theta),$$

since $a_1a_2 = \frac{l}{2}\left(1 - \cos\theta\right)$ and $M = ml$, l being the length of the needle. Therefore we have

$$\tfrac{1}{2}K\omega^2 = HM(1 - \cos\theta), \tag{vi}$$

in which θ is the angle through which the needle is thrown by the discharge Q.

Using equations (iv), (v), and (vi), the quantities G, $\frac{K}{M}$, and ω may be eliminated from (iii), giving, when solved for Q,

$$Q = \frac{2 I\tau}{\pi \tan \phi} \sin \tfrac{1}{2} \theta. \qquad (225)$$

This equation expresses Q in terms of easily observed quantities; viz., the permanent angle of deflection ϕ produced by a known current I, the time of vibration τ of the needle, and the angle θ through which the needle is thrown by the passage of the charge Q through the galvanometer.

If the value of Q is to be free from errors other than those involved in the determination of I, τ, ϕ, and θ, the conditions implied in the equations (iii), (iv), (v), and (vi) must be complied with. A galvanometer constructed with a view to the realization of these conditions is properly a ballistic galvanometer. These conditions are as follows:

Conditions implied by equation (iii).

(1) The needle must be at rest when the discharge through the coils begins.

(2) The directing field H must be in the plane of the coils.

(3) The field f due to the current in the coils must remain sensibly perpendicular to the axis of the needle during the entire discharge, so that the torque may be equal to GIM throughout the whole time of its action. This condition requires a slow vibration of the needle (τ large), except in cases when the duration of the discharge is very brief.

Conditions implied by equation (iv).

(1) The suspending fiber must be free from torsion.

(2) The damping must be so slight as to have no sensible influence upon the time of vibration of the needle.

Condition implied by equation (v).

The diameter of the coils must be large compared with the length of the needle; in other words, the law of the tangent galvanometer must hold true for such deflections as are produced by the current used in calibrating the instrument.

Condition implied by equation (vi).

The whole of the energy ($\frac{1}{2}K\omega^2$) must be employed in turning the needle against the directing field (H). This means that there must be no damping, a condition which cannot be realized in practice. For the failure to fulfil this condition corrections must be applied.

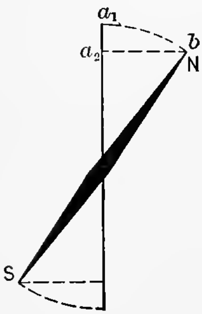

Fig. 194.

Working formulæ and approximate correction for damping. — For work not requiring extreme accuracy equation (225) may be simplified. Let a (Fig. 194) be the observed throw, in scale divisions, produced by the discharge. Let b be the permanent deflection, in scale divisions, produced by the current I, and let D be the distance, also in scale division, of the scale from the mirror. Then we have, approximately, $\sin\frac{1}{2}\theta = \frac{a}{4D}$, and $\tan\phi = \frac{b}{2D}$, whence equation (225) becomes

$$Q = \frac{I\tau a}{\pi b} \tag{vii}$$

When a vibrating body has reached its extreme position on either side of its position of equilibrium, it is said to be *in elongation*, and its distance (or angle) from the position of equilibrium is called *an elongation*. The successive elongations of a damped vibrating body form ordinarily a decreasing geometric series. Let k be the ratio of an elongation to the next following, then

$$k = \sqrt[m]{\frac{a_0}{a_m}}, \tag{viii}$$

wherein a_0 is an elongation, and a_m is the m^{th} following elongation. The quantity k is called the ratio of damping. Its natural logarithm is called the *logarithmic decrement* of the vibrations.

The observed throw a in equation (vii) is smaller in the ratio,

nearly, of $1:\sqrt{k}$ than it would be were there no damping, so that we should write $a\sqrt{K}$ for a in equation (vii), whence we obtain

$$Q = \frac{I\tau a\sqrt{k}}{\pi b}. \tag{226}$$

If k is very nearly unity, this equation leaves only very small outstanding reduction errors.

387. Comparison of two charges by means of the ballistic galvanometer. — Equation (226) may be written

$$Q = Ka, \tag{227}$$

in which K, written for $\frac{I\tau\sqrt{k}}{\pi b}$, is called the *reduction factor* of the galvanometer. If a second discharge, Q', be sent through the instrument, and the throw (a') be observed, we have

$$Q' = Ka'. \tag{227 bis}$$

Dividing these equations, member by member, we have

$$\frac{Q}{Q'} = \frac{a}{a'}, \tag{228}$$

which determines the ratio $\frac{Q}{Q'}$ independently of K.

The reduction factor of a ballistic galvanometer may be determined from equation (227), when the throw produced by a known charge has been observed.

388. Condensers; electrostatic capacity. — Kirchhoff's law, as stated in Art. 379, is strictly true only for established constant currents which can exist only in closed circuits.

It is necessary to consider also the flow of currents in circuits which are not closed. The simplest case of the flow of current in an open circuit is as follows: When the terminals of a bat-

tery B (Fig. 195) are connected to two large metal plates *cc cc*, a momentary current flows out of one plate through the battery and into the other plate, as indicated, and the plates become charged with electricity. If the plates thus charged are disconnected from the battery and connected by a wire through a ballistic galvanometer, the throw of the galvanometer will show that the charge which was collected upon *cc cc* is *proportional* to the e. m. f. of the battery; that is,

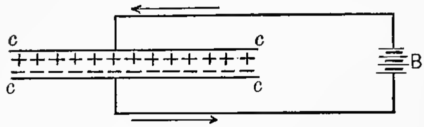

Fig. 195.

$$Q = JE, \tag{229}$$

in which Q is the charge which has collected upon the plates as measured by a ballistic galvanometer, E is the e. m. f. of the battery, and J is a constant for the given arrangement of metal plates. Two metal plates arranged in this way are called a *condenser*, and the quantity J is called its *electrostatic capacity*.

The plates *cc cc* may be separated by any substance which does not conduct the electric current; *e.g.* air, mica sheets, paraffined paper, or oil. Condensers will be further considered in Chapter IX.

389. The farad; the microfarad. — When Q and E in equation (229) are expressed in c. g. s. units, J is said to be expressed in c. g. s. units. When E is expressed in volts, and Q in coulombs, J is expressed in terms of a unit of capacity called the *farad*. Therefore a condenser has a capacity of one farad when it carries a charge of one coulomb with an e. m. f. of one volt between its plates. The farad is very large compared with the capacities of condensers ordinarily used in practice. The *microfarad*, one-millionth of a farad, is a more convenient unit.

390. Measurement of electrostatic capacity. — The electrostatic capacity of a condenser is easily determined by measuring the charge that it will carry with a measured e. m. f. between its plates. Equation (229) thus enables the calculation of J.

The instrument employed is the ballistic galvanometer. Indirect methods for measuring electrostatic capacity and methods for comparing electrostatic capacities have been devised by Maxwell and others.

Electrolysis and Batteries.

391. Electrolysis. — When the electric current passes through a conducting liquid which is not a chemical element, the liquid is decomposed. For example, molten NaCl is broken up into metallic sodium and chlorine by the electric current. Many liquids (for example, pure water, pure alcohol, etc.) scarcely conduct the electric current at all. When salts or acids are dissolved in such liquids they become conductors. In such cases it is the dissolved substance which is decomposed by the current. The products of decomposition of the dissolved substance sometimes react upon the solvent, however, thus decomposing it. For example, in an aqueous solution, H_2SO_4 is broken up into hydrogen and SO_4 by the current. The hydrogen appears at one electrode and escapes as a gas, and the SO_4 appears at the other electrode, where it acts upon the water, forming H_2SO_4, which goes into solution, and oxygen, which escapes as a gas.

The decomposition of a liquid by the electric current is called *electrolysis*, and liquids which are decomposed by the current are called *electrolytes*. Solutions of salts and acids generally are electrolytes. Electrolytes are mostly poor conductors (high resistance) compared with the metals, so that electrolysis is usually carried out in a vessel provided with two large conducting plates of metal or carbon called *electrodes* (Fig. 196). Such an arrangement is called an electrolytic cell. The electrode at which the current enters the cell is called the *anode*. The electrode at which the current leaves the cell is called the *cathode*.

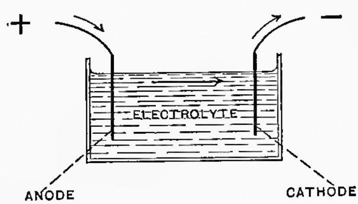

Fig. 196.

392. The laws of electrolysis. — *Faraday's first law. Electrochemical equivalent of a metal. The amount of metal deposited electrolytically by a current is proportional to the strength of the current and to the time.* We have, therefore,

$$M = KIt, \qquad (230)$$

in which M is the mass of metal, in grams, deposited in t seconds by a current I, and K is a constant for a given metal. This quantity K is called the *electrochemical equivalent* of the metal. The product It is the electric charge which has passed through the cell, so that Faraday's law may also be stated thus:

The deposit of metal is proportional to the charge which has passed through the electrolytic cell.

Ordinarily, electrochemical equivalents are specified in grams per coulomb.

Faraday's second law. — *The electrochemical equivalents of the various metals* (and of other elements which can form an ion of an electrolyte) *are proportional to the quotients of their atomic weights divided by their valencies.*

393. Table of electrochemical equivalents.

TABLE.

Element.	Valency.	Equivalent in grams per coulomb.	Element.	Valency.	Equivalent in grams per coulomb.
Aluminium,	III.	0.00009450	Potassium,	I.	0.0004054
Copper,	I.	0.00065420	Silver,	I.	0.001118
Copper,	II.	0.00032710	Sodium,	I.	0.0002387
Gold,	III.	0.00067910	Tin,	II.	0.0006116
Hydrogen,	I.	0.00001038	Tin,	IV.	0.0003058
Iron,	II.	0.0002909	Zinc,	II.	0.0003370
Iron,	III.	0.0001935	Bromine,	I.	0.0008282
Lead,	II.	0.001072	Chlorine,	I.	0.0003672
Magnesium,	II.	0.0001243	Iodine,	I.	0.001314
Mercury,	I.	0.002075	Nitrogen,	III.	0.00004850
Mercury,	II.	0.001037	Oxygen,	II.	0.00008287
Nickel,	II.	0.0003043			

394. The dissociation theory of electrolysis.—An electrolytic salt or acid when in solution, or when melted, is thought to be more or less dissociated into what are called its *ions*. For example, the ions of $CuSO_4$ are Cu (atoms) and SO_4; the ions of NaCl are Na (atoms) and Cl (atoms). These ions are supposed to be very highly charged with electricity, and to wander about through the solution. When an electric current passes through the electrolyte, the positively charged ions (cations) move towards the cathode, where they part with their positive charges and are deposited as hydrogen or metal, as the case may be; and the negatively charged ions (anions) move towards the anode, where they part with their negative charges. This movement of positively and negatively charged ions constitutes the electric current in the electrolyte.

All ions of a given substance have equal charges, so that to transfer a quantity *It* of electricity through the electrolytic cell, a proportional number of ions must move through the solution and be deposited upon, say, the cathode (*Faraday's I. Law*). The charge of an ion is proportional to its valency. Thus the copper ion in a solution of $CuSO_4$ has twice as much charge as the hydrogen ion, for example, in a solution of H_2SO_4. Half as many copper ions as hydrogen ions are deposited, therefore, by a given current in a given time. The atom of copper weighs 63.3 times as much as the atom of hydrogen, so that the electrochemical equivalent of copper is to the electrochemical equivalent of hydrogen as $\frac{63.3}{2} : 1$ (*Faraday's II. Law*).

A metal which has two valencies has also two distinct electrochemical equivalents. Thus ferric iron (valency, three) has an electrochemical equivalent two-thirds as large as ferrous iron (valency, two).

395. Hittorf's ratio.*—Consider an electrolyte, a solution of $CuSO_4$, for example. During electrolysis, this salt is in part decomposed, copper being deposited upon the cathode, and SO_4

* This is very closely related to what the Germans call Überführungszahl.

being liberated at the anode, so that, on the whole, the solution becomes less concentrated. Hittorf, taking precautions against the mixing of the solution by liquid currents and diffusion, found that this weakening of the solution of an electrolyte occurs wholly in the immediate neighborhood of the electrodes. Let a be the deficiency of $CuSO_4$* near the anode, and c the deficiency of $CuSO_4$ near the cathode. The ratio $\frac{a}{c}$, which is called *Hittorf's ratio, has a definite characteristic value for every electrolytic salt or acid* in dilute solution.

In most cases, *e.g.* in the electrolysis of $CuSO_4$, the anion reacts upon the solvent and goes into solution. The solution near the anode is then no longer a simple solution of the original salt, but contains, in addition, the products resulting from the breaking up of the anion, or the products resulting from the action of the anion upon the solvent or upon the material of the anode. Thus, if a copper anode is used in the electrolysis of $CuSO_4$, the anion SO_4 attacks the copper anode, forming $CuSO_4$, which goes into solution. In this case, the solution in the neighborhood of the anode would have an excess of $CuSO_4$ exactly equal to the deficiency of $CuSO_4$ in the neighborhood of the cathode. In copper plating, a copper anode is used, in order that fresh $CuSO_4$ may be thus formed continually.

This weakening of an electrolyte is easily shown by the upward streaming of the weakened solution along the faces of the electrodes. In case the solution becomes denser because of secondary reactions, as near a copper anode in a solution of $CuSO_4$, the solution will be seen to stream downwards.

396. Proposition. — Hittorf's ratio for a given salt or acid is equal to the ratio of the velocities of the cations and anions, respectively, as they move through the electrolyte carrying the current.

* That is, a deficiency *of the salt* independent of any reaction at the electrode which may produce the salt.

Proof. * — Consider the electrolysis of a solution of $CuSO_4$. Suppose the electrolysis to have continued until 159.3 grams of $CuSO_4$ (one gram-molecule) have been decomposed, 63.3 grams of copper being deposited on the cathode, and 96 grams of SO_4 being liberated at the anode.

If we imagine the current through the electrolyte to depend entirely upon the movement of copper ions, the SO_4 ions being supposed stationary throughout the middle portions of the electrolyte, then the solution near the anode will have become deficient in $CuSO_4$ by the whole amount of 159.3 grams, and the solution will be unchanged in strength everywhere else.

If we imagine the whole current through the electrolyte to depend entirely upon the movement of SO_4 ions, the copper ions being supposed stationary throughout the middle portions of the electrolyte, then the solution near the cathode will have become deficient in $CuSO_4$ by the whole amount of 159.3 grams, and the solution will be unchanged in strength everywhere else.

If the copper ions move n times as fast as the SO_4 ions through the electrolyte, then $\frac{n}{n+1}$ of the current may be thought of as due to the movement of copper ions, and $\frac{1}{n+1}$ of the current may be thought of as due to the movement of SO_4 ions.

On account of the movement of copper ions the solution in the neighborhood of the anode will have become deficient in $CuSO_4$ by the amount of $\frac{n}{n+1} \times 159.3$ grams, since 159.3 is the deficiency at the anode which would be produced if the whole current were due to the movement of copper ions.

Similarly, $\frac{1}{n+1} \times 159.3$ grams is the deficiency in $CuSO_4$ in the neighborhood of the cathode, on account of the movement

* This is not strictly a *proof*, but an application of the dissociation theory in the interpretation of an experimental fact, viz. that Hittorf's ratio is a constant for a given substance.

of the SO_4 ions. Therefore the ratio of these deficiencies is n. Q.E.D.

It is to be kept in mind in this discussion that as the SO_4 ions move through the electrolyte away from the vicinity of the cathode, the same number of Cu ions are deposited at the cathode without having to travel through the middle portions of the electrolyte. The same is true of the liberation of SO_4 ions at the anode as the copper ions move away from the vicinity of the anode towards the cathode. Thus if the copper ions and SO_4 ions were to move through the solution at the same velocity, half of the copper which is deposited on the cathode would travel through the solution from the vicinity of the anode, and half would come from the immediate vicinity of the cathode, because of the movement of SO_4 ions away from that region towards the anode.

397. Hittorf's numbers.* — Let the velocity of hydrogen ions under given conditions be taken as unity. Then the velocity of SO_4 ions under the same conditions is found, by multiplying this unit velocity by Hittorf's ratio for H_2SO_4. The velocity of copper ions may then be found by dividing the velocity of SO_4 ions by Hittorf's ratio for $CuSO_4$, the velocity of chlorine ions by multiplying the unit velocity of hydrogen ions by Hittorf's ratio for HCl, etc. In this way values may be calculated for the relative ionic velocities of various substances. These relative ionic velocities are called *Hittorf's numbers.*

398. Molecular conductivity of electrolytes; ratio of dissociation. — Let c be the concentration of an electrolyte in gram-molecules † of salt per liter of solution. The number of actual

* German; Wanderungsgeschwindigkeiten.

† The "gram-molecule" is a number of grams of a salt equal numerically to its molecular weight. Thus one gram-molecule of $NaCl$ is 58.5 grams of $NaCl$. One gram-molecule of KCl is 74.6 grams of KCl. There is, of course, the same number of actual molecules of $NaCl$ in 58.5 grams of that salt as there is of KCl in 74.6 grams of that salt.

molecules of the salt per cubic centimeter of solution is proportional to c. Let $1:\mu$ be the ratio of the whole number of molecules of salt per cubic centimeter of solution, to the number of molecules which have been dissociated into ions. The number of ions per cubic centimeter of solution is then proportional to μc.

The amount of current carried by a given electrolyte is proportional to the number of ions per c.c., everything else remaining the same. — But anything which increases current with constant e. m. f. must decrease resistance or increase conductivity in the same proportion. Therefore the specific conductivity, k, of a given electrolyte is proportional to μc. In very dilute solutions, μ approaches unity; *i.e.* all the molecules of the salt are dissociated, so that μc then becomes c, and k is then proportional to c. The ratio $\frac{K}{c}$ is then a constant. This constant is called the *molecular conductivity* of the electrolyte. Representing the molecular conductivity by m, we have, for very dilute solutions, $\frac{K}{c} = m$, or for solutions of ordinary concentration $\frac{K}{\mu c} = m$, or:

$$K = \mu c m. \tag{231}$$

The values of K and c are easily determined in every case, the value of m is the ratio $\frac{K}{c}$, when the solution is very dilute, and is thus easily determined. Therefore the ratio of dissociation, μ, of a given electrolyte may be calculated when m has been determined in this way for a dilute solution of the electrolyte and the values of K and c have been determined for the given solution. There is an independent method for determining the values of μ; namely, by observing the freezing point of the electrolyte. This method gives in every case the same values of μ as the electrical method.

The molecular conductivities, at a given temperature, of various electrolytes are proportional to the sums of the Hittorf's numbers, at that temperature, for the ions of the respective electrolytes.

399. Work spent in forcing an electric current through an electrolytic cell. — This work consists mainly * of three parts.

(a) The work which appears as heat throughout the electrolyte. The rate at which work is so spent, or the rate of generation of heat throughout the electrolyte, is accurately *proportional to the square of the current.* The proportionality factor, R, is called the resistance of the electrolyte.

(b) The work which appears as heat *at the electrodes* on account of the sweeping process (see Art. 276, Vol. I.) which takes place there as the ions are deposited. The rate at which work is so spent increases very rapidly for a few moments after the current is started, and then remains nearly constant, for a given value of the current, etc. This rate of expenditure of work is thus practically a function $\phi(i)$, of the current after the current is once well established.

(c) The chemical decomposition of the electrolyte by the current *requires* work, and the reaction of the liberated ions upon the solvent or upon the electrodes is often a *source* of work. The net rate at which work is so spent is accurately *proportional to the current;* it may be either *positive* or *negative* according as the work required to decompose the dissolved salt or acid is *greater* or *less* than the work generated by the action of the liberated ions upon the solvent and upon the electrodes.

400. Energy equation of an electrolytic cell. — Let a current i be forced through an electrolytic cell by an outside agent (battery or dynamo) of which the e. m. f. is E. The rate at which this agent does work upon the cell is then Ei, from equation (219). The rate at which energy appears as heat throughout the electrolyte is Ri^2. The rate at which energy appears as heat at the electrodes, provided the current has been flowing for some time, is $\phi(i)$. Finally, the rate at

* We here ignore thermo-electromotive forces, as being in every case inconsiderable; and e. m. f.'s at places where the electrolyte changes in concentration or composition as carrying the subject beyond limits.

which energy is used in bringing about the chemical action is ei. We have, therefore,

$$\left.\begin{aligned} Ei &= Ri^2 + \phi(i) + ei, \\ \text{or} \qquad E &= Ri + \frac{\phi(i)}{i} + e. \end{aligned}\right\} \tag{232}$$

in which R is the resistance of the electrolytic cell, *i.e.* the proportionality constant mentioned in item (a) Art. 399; e is the proportionality constant mentioned in item (c) Art. 399, and $\phi(i)$ is the function implied in item (b) Art. 399.

The second form of equation (232) shows that the e. m. f. E, of the agent, may be thought of as broken up into three parts, viz.: the part Ri, which is used to overcome the resistance of the electrolyte; the part e, which balances what is called the *counter e. m. f.* of the electrolytic cell; and the part $\frac{\phi(i)}{i}$, which overcomes what is sometimes called the *polarization* of the cell.

The polarization $\frac{\phi(i)}{i}$ of an electrolytic cell manifests itself at both electrodes. At each electrode the polarization depends upon the *current per unit area of the electrode*, or, in other words, upon what is called the current density.

Remark. — When the current i, equation (232), is very small, the term Ri becomes negligible in comparison with e and E; but the term $\frac{\phi(i)}{i}$ ordinarily remains finite. The sum $\frac{\phi(i)}{i} + e$, when i is small, is the least value of the e. m. f. E that will suffice to send current through the electrolytic cell.

401. *Example.* — The passage of current through an electrolytic cell containing H_2SO_4 results in the generation of oxygen and hydrogen, so that the chemical action produced is in effect the decomposition of H_2O. The decomposition of one gram of H_2O requires 162×10^9 ergs of energy. A current of strength i c. g. s. units, decomposes $0.000933\,i$ gram of water per second, requiring the expenditure of $150 \times 10^6 i$ *ergs per second*. This

is equal to ei by Art. 399. Therefore e is in this case 150×10^6 c. g. s. units of e. m. f., or 1.5 volts. The *least* e. m. f. which will produce a current through such an electrolytic cell is found by observation to be about 2.1 volts. Therefore the minimum value of $\frac{\phi(i)}{i}$ is in this case about 0.6 volt. As the current through such a cell increases, Ri increases in proportion, and the electromotive force $\frac{\phi(i)}{i}$ required to overcome polarization increases greatly. Figure 197 represents graphically the observed values of $E\left(= Ri + \frac{\phi(i)}{i} + e\right)$ for various values of i for an electrolytic cell containing a twenty per cent solution of H_2SO_4, with flat electrodes 16 cm. × 21 cm. and 6 cm. apart. The ordinate of the horizontal dotted line represents the constant value of e, the distances from the horizontal dotted line to the inclined dotted line represent the values of Ri, and the distances from the inclined dotted line to the full-line curve represent the values of $\frac{\phi(i)}{i}$. No doubt the resistance of such an electrolytic cell increases slightly as the current increases, because of the choking of the electrolyte by bubbles of gas.

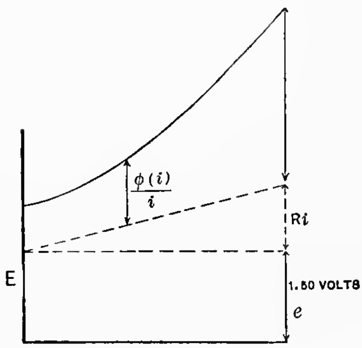

Fig. 197.

402. Voltaic batteries. — An electrolytic cell, in which the chemical actions brought about by the current are a *source* of energy, is called a *voltaic battery*. For example, an electrolytic cell containing dilute H_2SO_4, and having a zinc anode, helps to maintain a current which passes through it, because of the fact that in the decomposition of H_2SO_4 less energy is required than is furnished by the combination of the SO_4 ions with the zinc of the anode. When the electrodes of such an electrolytic cell, or

battery as we shall henceforth call it, are connected by a wire, a current is started and maintained without the aid of any external agent.

403. Energy equation of a voltaic battery. — Since E is zero, equation (232) becomes for this case

$$\left.\begin{aligned} ei &= Ri^2 + \phi(i), \\ \text{or} \qquad e &= Ri + \frac{\phi(i)}{i}, \end{aligned}\right\} \tag{233}$$

in which ei is the rate at which energy is *furnished* by the chemical actions in the battery, Ri^2 is the rate at which energy appears as heat throughout the circuit, including electrolyte and wire connecting the electrodes, and $\phi(i)$ is the rate at which energy appears as heat at the electrodes, on account of the sweeping processes which take place there, as the ions are deposited.

Solving equation (233) for i, we have

$$i = \frac{e - \dfrac{\phi(i)}{i}}{R}. \tag{234}$$

The quantity $e - \dfrac{\phi(i)}{i}$ is the electromotive force of the battery. The quantity e is the electromotive force that the battery would have if all the energy of the chemical reactions were available in producing current; that is, if it were not for the sweeping processes at the electrodes. These processes result in the conversion of energy into heat at the rate $\phi(i)$, and they thus reduce the effective e. m. f. of the cell by the amount $\dfrac{\phi(i)}{i}$, which is called the *polarization* of the battery. The quantity e is numerically equal to the ergs of energy developed by the chemical action which takes place during one second when one unit of current is flowing through the cell. This energy may be measured as heat if the given chemical action is made to take place in a calorimeter and thus e may be determined.

For some batteries the polarization $\frac{\phi(i)}{i}$ is quite small when the current is very small. The e. m. f. $\left(e - \frac{\phi(i)}{i}\right)$ of such batteries, when giving a small current, being sensibly equal to e, may be calculated from thermo-chemical data, as explained above.

404. *Example.* — Figure 198 represents graphically the electromotive force $e - \frac{\phi(i)}{i}$ of a Leclanché battery (see 406, (*c*)) for different values of the current. The values of $e - \frac{\phi(i)}{i}$, being

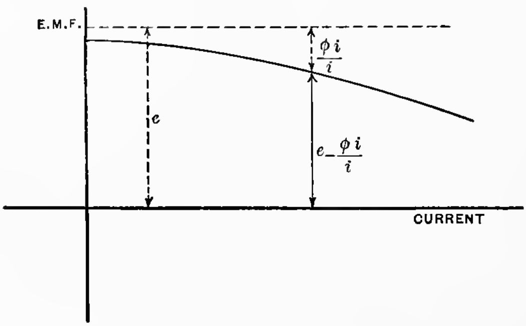

Fig. 198.

equal to Ri by equation (233), in which R is the total resistance of the circuit, were determined by measuring i for different values of R. Each value of current was allowed to flow for two minutes so that the polarization might reach its maximum value for that value of the current. The ordinate of the dotted line represents approximately the constant value of e, and the distance from the dotted line to the full-line curve represents the value of the polarization $\frac{\phi(i)}{i}$.

Figure 199 represents the growth of the polarization immediately after a current of 5 amperes was started in a Grenet battery. The current was kept constant by varying the resistance of the circuit. The e. m. f. of the battery (e — polarization) was

determined by adding, to the measured e. m. f. between the terminals, the portion Ri which is used to overcome the resistance of the electrolyte, R being the resistance of the battery.

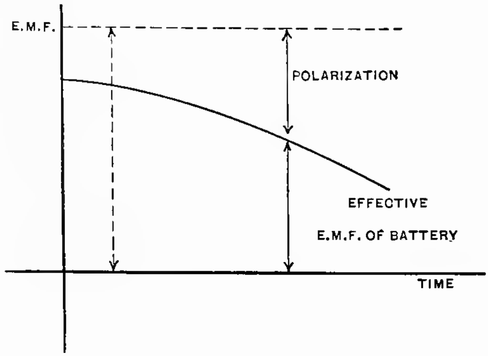

Fig. 199

405. Local action. — In some forms of battery, chemical action goes on independently of the flow of current, when no current is flowing as well as when current is flowing. Such chemical action contributes, in no way, to the production of current by the battery, and it is called *local action.* For example, in a Grenet cell, even when it is being used to furnish a large current, as much as twenty per cent of the zinc and acid used is lost in local action.

406. Forms of batteries. — (*a*) The *simple voltaic* cell (Fig. 200) consists of an anode of zinc and a cathode of copper (or any metal not attacked by the acid) immersed in dilute H_2SO_4. When the current flows, the hydrogen ions appear at the copper plate, and pass off as a gas, and the SO_4 ions, appearing at the zinc plate, react upon it, forming $ZnSO_4$ which goes into solution. The chem-

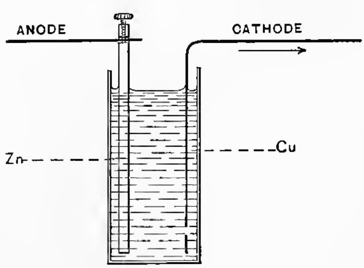

Fig. 200.

ical action which takes place in this cell is represented by the equation

$$Zn + H_2SO_4 = ZnSO_4 + 2\,H.$$

The net energy of this 'reaction' is small, so that the value of e, of equation (233), is small. The polarization $\frac{\phi(i)}{i}$ of this cell is considerable, particularly when it is giving a large current, and the e. m. f. $\left(e - \frac{\phi(i)}{i}\right)$ of the cell ranges from near one volt, with zero current, down to half a volt when the current is large.

(*b*) **The Bunsen, Grove, and Grenet cells.** — If the cathode of a simple cell be made of carbon or platinum, it may be submerged in an active, oxidizing agent such as nitric acid or chromic acid. The strong acid is prevented from mixing with the dilute sulphuric acid surrounding the zinc anode by means of a containing cup of porous earthenware. This porous earthenware does not break the continuity of the electrolyte. When the current flows, SO_4 ions appear at the zinc anode, forming $ZnSO_4$, and the hydrogen ions appear at the cathode, and are oxidized, forming water. The available energy of this reaction is quite considerable, giving a battery having a large value of e; also, the polarization of such a cell is not very great even with large currents. Such a cell has an e. m. f. $\left(e - \frac{\phi(i)}{i}\right)$ of about 2.25 volts.

When nitric acid is used with a carbon cathode, the cell is called a *Bunsen cell*. When platinum is substituted for carbon, the cell is called a *Grove cell*. In the *Grenet cell*, chromic acid is used.

The presence of chromic acid in the solution, near the zinc, does not lead to a hurtful amount of local action, so that, for convenience, the porous cup is often dispensed with in the Grenet cell.

(*c*) **The Leclanché cell.** — This cell has an anode of zinc and a cathode of carbon mixed with powdered MnO_2, immersed in a solution of NH_4Cl. When the current flows, Cl ions appear at

the zinc anode, and form $ZnCl_2$. The NH_4 ions appear at the cathode, break up into NH_3 and H; the NH_3 goes into solution, and the H combines with oxygen furnished by the MnO_2. The e. m. f. of this cell is about 1.6 volts with a small current. It

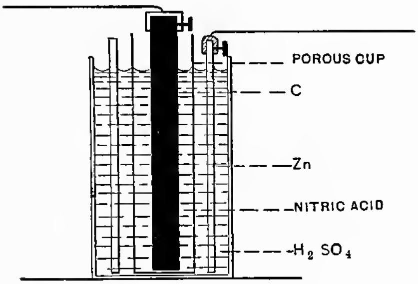

Fig. 201.

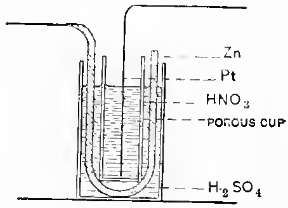

Fig. 202.

polarizes considerably with large currents. The valuable feature of this cell is its entire freedom from local action.

Figures 201, 202, 203, and 204 show the usual forms of the above-mentioned cells.

(*d*) **Daniell cell.** — This cell has an anode of zinc immersed in a solution of $ZnSO_4$, or in dilute H_2SO_4, and a copper cathode immersed in a solution of $CuSO_4$. The two solutions are kept from mixing by a variety of devices, some of which are described below. When the current flows, SO_4 ions appear at the zinc anode, forming $ZnSO_4$, and copper ions appear at the cathode, and are deposited as metallic copper. The chemical reaction in this cell is $Zn + CuSO_4 = Cu + ZnSO_4$. This cell is almost free from polarization. It has an e. m. f. of about 1.08 volts.

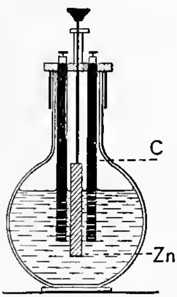

Fig. 203.

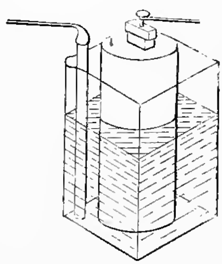
Fig. 204.

In the original form of Daniell cell, a cylindrical jar of copper, the inner surface of which served as the cathode, contained a

porous cup (Fig. 205), within which was placed a rod of zinc. The solution of $CuSO_4$ within the jar was continually replenished from some immersed crystals which were held within a

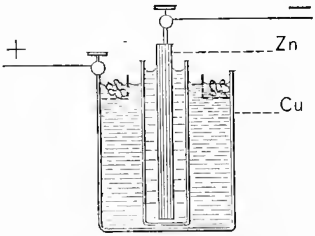

Fig. 205.

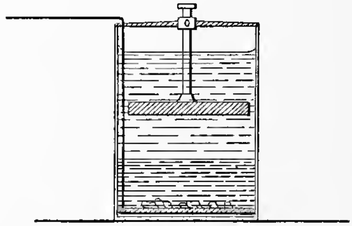
Fig. 206.

pocket of perforated copper foil. The porous cup was filled with dilute H_2SO_4, which soon became converted into $ZnSO_4$.

The *gravity battery* (Fig. 206) is a Daniell cell in which the concentrated solution of $CuSO_4$ occupies the lower portion of the containing vessel, and the less dense solution of $ZnSO_4$ floats on top of it.

Fleming, in a standard cell based upon the principle of the Daniell battery, used the device shown in Fig. 207 to prevent the two liquids from mixing. Rods of zinc and of copper respectively are inserted in the two arms of a U-shaped tube. The solution of $ZnSO_4$ was then introduced through the stopcock *A*, and of $CuSO_4$ through *B*. The line of demarkation at the level of the stopcock *C*, could be freshly reëstablished whenever the diffusion of the two liquids had begun to modify the action of the cell.

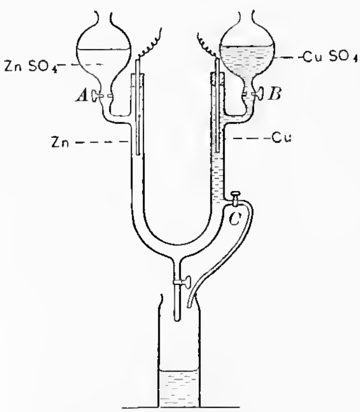

Fig. 207.

(*c*) **Clark's standard cell.** — This cell has a cathode of mercury, over the surface of which is spread a paste made by

rubbing up powdered mercurous sulphate with a concentrated solution of zinc sulphate. Over this paste is a clear concentrated solution of zinc sulphate, in which is the zinc anode. The connection to the mercury cathode is often made by means of a platinum wire passing through the glass containing vessel. When current flows through this cell, SO_4 ions appear at the zinc anode, forming $ZnSO_4$, and Zn ions appear at the mercury cathode and react upon the mercurous sulphate, forming $ZnSO_4$ and metallic mercury. This cell polarizes very much when any considerable current flows through it. With small currents, however, its e. m. f. is very nearly constant, varying slightly with temperature. It serves admirably as a standard of e. m. f. The arrangement of the Clark cell, devised by Lord Rayleigh and used by him in his researches upon this form of battery, is shown in Fig. 208.

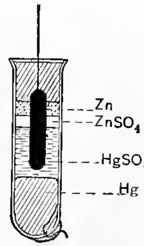

Fig. 208.

407. Storage batteries. — The chemical action which takes place in a battery when it is giving current is exactly reversed when current is forced through it in the opposite direction by an external agent. Any battery, therefore, may in this way be regenerated to a greater or less extent after it has been used for some time in the production of current. In order that the regeneration of the battery may be complete, it is necessary that the battery be free from local action, and that the products resulting from the chemical action which takes place when the cell is producing current be conserved in the electrolyte. A battery in which these conditions are realized is called a *storage battery*.

408. The lead storage battery. — The electrodes of this battery consist of two massive lead grids, the meshes or interstices of which are filled with a paste of $PbSO_4$ made by mixing litharge (PbO), or litharge and red lead (PbO_2), with dilute

sulphuric acid. Figure 209 shows one of the numerous forms of grid. These electrodes are placed in a dilute solution of H_2SO_4. When a current is sent through the cell hydrogen ions appear at an electrode, react upon the $PbSO_4$, forming spongy Pb and H_2SO_4 which goes into solution, and SO_4 ions appear at the electrode and produce the following reaction:

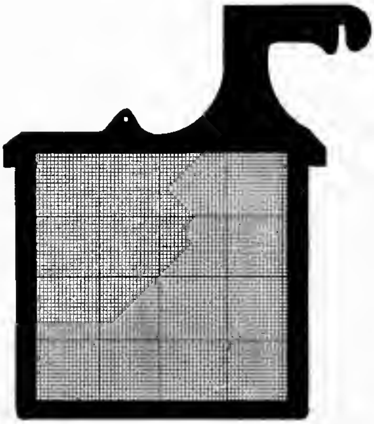

Fig. 209.

$$PbSO_4 + SO_4 + 2\ H_2O = PbO_2 + 2\ H_2SO_4.$$

This H_2SO_4 goes into solution, and the peroxide of lead, PbO_2, is left on the grid. When all or most of the $PbSO_4$ on the electrodes has been changed in this way to Pb and PbO_2, respectively, the cell is said to be charged, and it may be used as an ordinary battery, giving a current in the opposite direction to the charging current, until most of the Pb and PbO_2 are changed back to $PbSO_4$. Such a battery gives upon discharging very nearly as many ampere-hours as is used in charging. But the polarization $\frac{\phi(i)}{i}$ being always opposed to the current, causes the e. m. f. of the battery to range from 2 volts to 2.3 as the battery is being charged, and from 2.1 to 1.8 volts as the battery is discharged, so that the energy efficiency of the cell is ordinarily about 80 per cent. Figure 210 shows graphically the growth of e. m. f. of an 80 ampere-hour cell while being charged with 10 amperes of current, and the decay of e. m. f. of the same battery while being discharged with the same current.

The active material ($PbSO_4$ paste) in the grid of a storage battery suffers considerable expansion and contraction during

charge and discharge. For this reason, mainly, it is necessary to charge the battery always in the same sense. Even then the

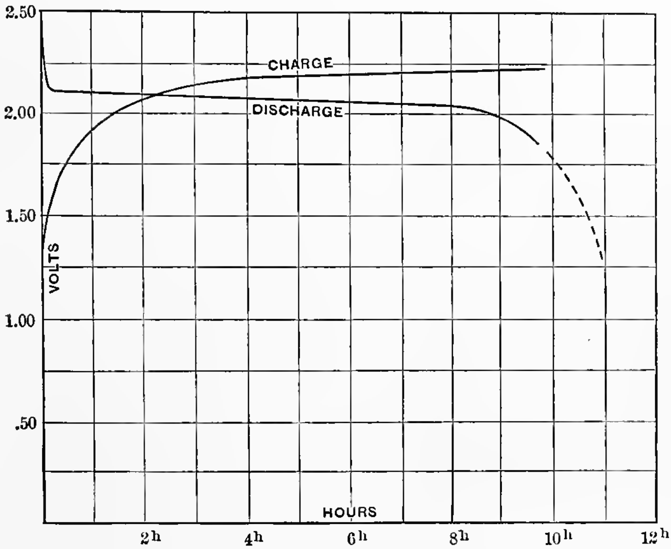

Fig. 210.

grid which is the anode during charge, and the cathode during discharge (the positive grid), tends to fall to pieces. The grids must therefore be repaired or replaced from time to time.

CHAPTER VI.

INDIRECT METHODS OF MEASURING CURRENT, RESISTANCE, ELECTROMOTIVE FORCE, AND THE MAGNETIC FIELD.

CURRENT.

409. The measurement of current by electrolysis. — The electrochemical equivalent of a metal having been determined, the strength of a current, which is made to deposit the metal electrolytically, may be calculated from the equation

$$M = kIt. \qquad (230\ bis)$$

In which equation, M is the weighed amount of metal deposited by the current I in t seconds. An apparatus for carrying out this method for measuring current is called a **voltameter.**

The electrolytic reactions which have been found most suitable for the measurement of current are: the decomposition of a silver salt with deposition of silver, the decomposition of a copper salt with deposition of copper, and the decomposition of water under conditions which admit of the collection and determination of the oxygen and hydrogen set free. We have, therefore, in common use:

(1) The silver voltameter.

(2) The copper voltameter.

(3) The water (or sulphuric acid) voltameter.

410. The silver voltameter. — In its usual form, this instrument consists of a platinum bowl containing an aqueous solution of pure silver nitrate. The current enters the voltameter through a plate of silver, — the anode (Fig. 211).

The apparatus is given in the form shown in the figure because the deposited silver, which is of crystalline structure, is not firmly adherent. The anode likewise suffers disintegration under the action of the current, and it is necessary to surround it with some porous septum, such as a layer of filter paper which catches the detached particles, and prevents their falling into the platinum bowl.*

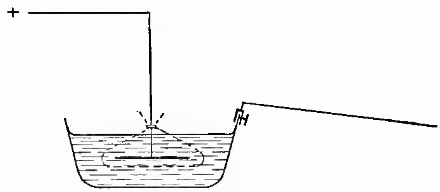

Fig. 211.

The bowl is accurately weighed before and after the operation, and its increase in mass measures the quantity M of the equation (230).

The electrochemical of silver is better known, perhaps, than that of any other metal, for which reason the silver voltameter is given the first place among the instruments of its kind. *One ampere of current deposits silver at the rate of* 0.001118 *gram per second.*

411. The copper voltameter.—This apparatus consists of a glass vessel containing an aqueous solution of copper sulphate and electrodes of metallic copper. The current, in its passage, removes copper from the anode, and deposits copper upon the cathode. The latter is weighed before and after the deposition. A form of copper voltameter which gives excellent results is

* According to the specifications of the Chamber of Delegates of Chicago Electrical Congress (1893), the arrangement for the measurement of about 1 ampere of current should be as follows:

(1) The platinum bowl should be not less than 10 cm. in diameter, and 4 or 5 cm. in depth.

(2) The anode should be a plate of pure silver 30 sq. cm. in area, and 0.2 or 0.3 cm. thick; it should be enveloped in white filter paper.

(3) The liquid should be a neutral solution of pure silver nitrate, about 15 parts by weight of the nitrate to 85 parts of water.

—Proceedings of the International Electrical Congress held in Chicago, 1893, p. 20.

shown in Fig. 212. It is called the *spiral coil voltameter.** The electrodes are spiral coils of copper wire. The inner coil is the cathode. The density of the solution should be about 1.15.

One ampere of current deposits 0.000328 gram of copper in 1 second.†

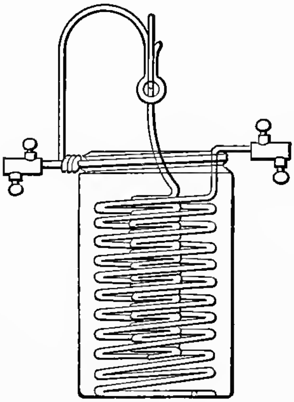

Fig. 212.

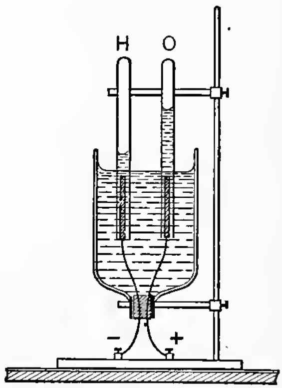

Fig. 213.

412. The water (or sulphuric acid) voltameter. — This instrument consists of a glass vessel containing dilute sulphuric acid. The electrodes are strips of platinum. The current liberates oxygen at the anode and hydrogen at the cathode. The gases are collected in separate vessels, as shown in Fig. 213, or in a common receiver, in which case the voltameter is given the form indicated in Fig. 214. Usually, the volumes of the liberated gases are measured, but the German chemist Bunsen, in whose hands the most accurate results have been obtained with the water voltameter, recombined the gases by means of the electric spark, and weighed the water thus formed.

One ampere of current liberates 0.0000104 gram of hydrogen and 0.0000828 gram of oxygen in 1 second.

* See Ryan, Transactions of the American Institute of Electrical Engineers, Vol. VI. p. 322.

† See further, Nichols, Laboratory Manual, Vol. I. p. 166.

413. Measurement of current by the potential method. — The current to be measured is passed through a known resistance R,

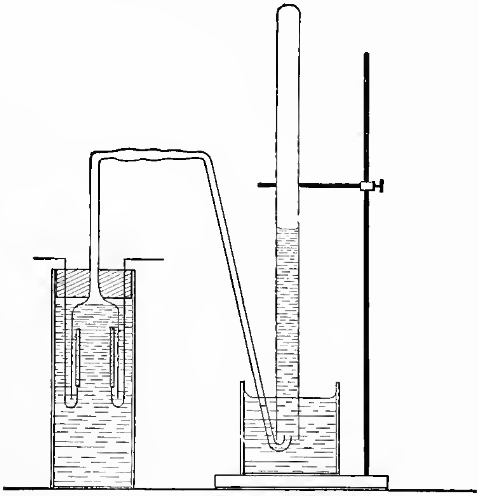

Fig. 214.

and the e. m. f. E, between the terminals of this resistance, is measured. (See Art. 423.) Then

$$I = \frac{e}{R}. \qquad (217\ bis)$$

414. Measurement of current by the electro-calorimeter. — The current to be measured is passed, for t seconds, through a known resistance which is submerged in an electro-calorimeter. (See Art. 373.) The rise in temperature of the calorimeter is observed, from which the total amount of heat H generated is determined. Then

$$H = RI^2t, \qquad (213\ bis)$$

This equation then enables the calculation of I.

415. Measurement of current by means of ammeters. — An ammeter is a galvanometer with a pointer playing over a scale which is constructed to give current directly. Most ammeters

are similar in principle either to the tangent galvanometer, the electrodynamometer, or to the D'Arsonval galvanometer. There are, however, two other distinct types, namely,—

The *plunger ammeter*, which depends upon the drawing of a soft iron plunger into a coil through which the current flows, the attraction of the coil being counteracted by a spring. A pointer attached to the plunger plays over a calibrated scale.

The *hot wire ammeter*, which depends upon the rise in temperature and consequent expansion of a wire through which the current passes. The expanding wire actuates an index which plays over a calibrated scale.

Resistance.

416. Resistance boxes or **rheostats** are arrangements by means of which wire of any desired resistance may be introduced into a circuit. The usual construction is as follows: A series of massive metal blocks are connected by wires whose resistances are 1, 2, 3 ohms, etc., respectively. The arrangement is shown in Fig. 215. By means of metal plugs which fit snugly between the blocks the blocks may be connected at pleasure, leaving the resistance between them practically zero.

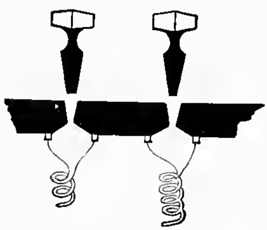

Fig. 215.

The word *rheostat* is perhaps oftener applied to arrangements for varying the resistance of a circuit by consecutive infinitesimal gradations. The wire which is interpolated in the circuit in this case is exposed. One end is connected permanently to the circuit, and any amount may be put into circuit by moving a *sliding contact* along the wire.

417. Measurement of resistance by substitution.—The resistance to be measured is connected in circuit with a battery and a galvanometer. The deflection of the galvanometer is

observed. The unknown resistance is then removed from circuit and a resistance box is put in its place. Plugs are then removed from the box until the galvanometer deflection is the same as before. The box reading is then the value of the unknown resistance.

418. By means of the tangent galvanometer. — Let it be required to determine the resistance of a battery, the connecting wires, and a tangent galvanometer. These are connected in circuit together with a resistance box. With all the plugs in the box the galvanometer deflection ϕ is observed. The current in the circuit is then equal to $K \tan \phi$. It is also equal to $\frac{E}{R}$ (equation (217)), E being the e. m. f. of the battery and R the total resistance of the circuit. Therefore

$$\frac{E}{R} = K \tan \phi. \tag{i}$$

By means of the box a known resistance a is added to the circuit and the galvanometer deflection ϕ' is again observed. The current is now $\frac{E}{R+a}$, so that

$$\frac{E}{R+a} = K \tan \phi'. \tag{ii}$$

Dividing equation (i) by (ii), member by member, we get

$$\frac{R+a}{R} = \frac{\tan \phi}{\tan \phi'}, \tag{iii}$$

from which R may be calculated.

419. Measurement of resistance by means of the differential galvanometer. — The differential galvanometer consists of a magnet, with attached mirror or pointer, suspended between two similar coils. The distances of these coils from the needle are adjusted until the *same* current flowing in opposite directions in these coils gives no deflection of the suspended magnet.

If each of the coils of this galvanometer is connected with a distinct circuit, then when the galvanometer gives no deflection the currents in the two circuits must be equal.

The following is the method of using such a galvanometer for the measurement of resistance:

A battery circuit branches at C and D (Fig. 216). One of these branches includes an unknown resistance x and one coil A of a differential galvanometer. The other branch includes a resistance box and the other coil B of the differential galvanometer. Plugs are removed from the box until the galvanometer gives no deflection. The unknown resistance x is then equal to the box reading.

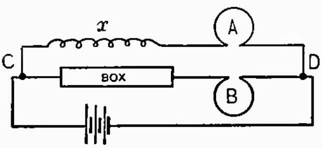

Fig. 216.

Proof. — The currents in the two branches being equal — as per indication of the differential galvanometer — the resistances of the branches must be equal (from equation (220)). The coils A and B being constructed to have the same resistance, the remaining resistances in the respective branches must be equal.

420. Measurement of resistance by means of Wheatstone's bridge. — Wheatstone's bridge consists of a network of conductors, as shown in Fig. 217. A battery circuit branches at a and b, the current flowing through resistances α, β, γ, and δ, as shown. A sensitive galvanometer is connected between c and d.

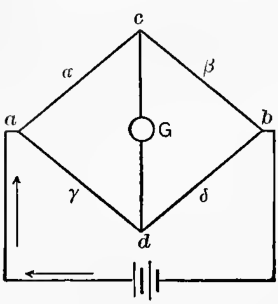

Fig. 217.

Proposition. — When no current flows through the galvanometer then

$$\frac{\alpha}{\beta}=\frac{\gamma}{\delta}. \tag{235}$$

Proof. — Let i' be the current flowing through α and β, — the same current flows through α and β since the galvanometer

current is zero, — and let i'' be the current flowing through γ and δ. The e. m. f. between c and d is zero, therefore the e. m. f. $\alpha i'$ between a and c is equal to the e. m. f. $\gamma i''$ between a and d; that is

$$\alpha i' = \gamma i'', \tag{i}$$

similarly $$\beta i' = \delta i''. \tag{ii}$$

Dividing (i) by (ii), member by member, we have $\frac{\alpha}{\beta} = \frac{\gamma}{\delta}$.

Q.E.D.

421. The slide-wire bridge is a form of the Wheatstone bridge. A stretched wire ab (Fig. 218), an unknown resistance α, a known resistance β, and a sensitive galvanometer g are connected as shown. The lettering in Fig. 218 corresponds to that in Fig. 217. The sliding contact d is moved along the wire until the galvanometer gives no deflection. Then $\frac{\alpha}{\beta} = \frac{\gamma}{\delta}$, from equation (235). But $\frac{\gamma}{\delta}$ is equal to the ratio of the lengths of the corresponding portions of the wire ab, and is thus easily determined, so that β being known α may be calculated. The usual arrangement of the slide-wire bridge is shown in Fig. 219.

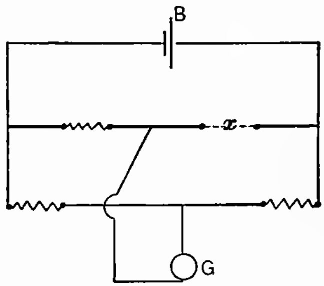

Fig. 218.

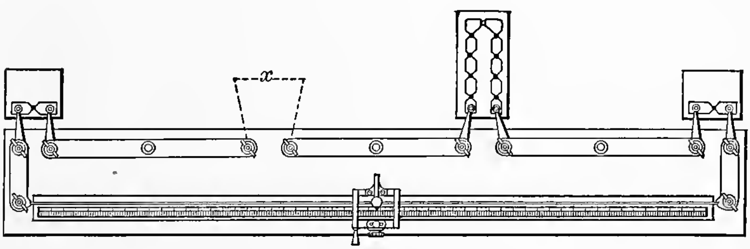

Fig. 219.

422. The box bridge is a resistance box containing three sets of resistances β, γ, and δ, connected as shown in Fig. 220. The dotted lines represent connections outside the box. The portions γ and δ of the box usually have each 10 *ohm*, 100 *ohm*, 1000 *ohm* plugs, so that the ratio $\frac{\gamma}{\delta}$ has a number of values which may be chosen at convenience. The portion β is an ordinary set of resistances. An unknown resistance α is connected as shown, the ratio $\frac{\gamma}{\delta}$ is chosen, and the value of β is changed until the galvanometer gives no deflection. The value of α is computed from equation 235.

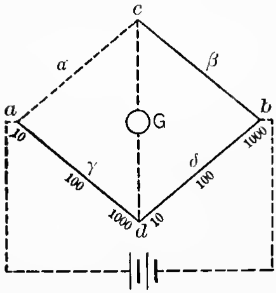

Fig. 220.

In Fig. 221 the arrangement of one of the best known forms of box bridge is shown. In this apparatus, which is known as the Anthony bridge, there are four sets of bridge coils corresponding to δ, γ of Fig. 220. The comparison coils corresponding to β are arranged so as to permit of the greatest possible

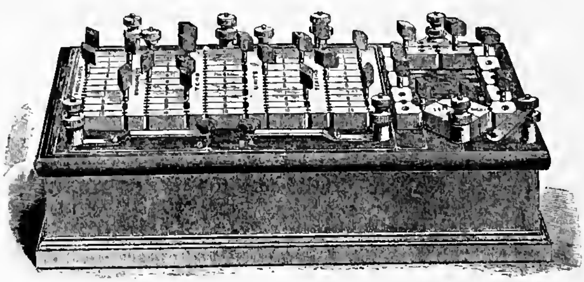

Fig. 221.

freedom of combination by the use of plugs between the parallel rows of metal blocks. There are ten coils of 1 ohm, ten of 10 ohms, ten of 100 ohms, ten of 1000 ohms, and ten of 10,000 ohms each. These may be combined so that each group of ten

becomes equivalent to a single coil of tenfold resistance or of $\frac{1}{10}$ the resistance of each member of the group.

Electromotive Force.

423. Measurement of electromotive force by the galvanometer. — Let I be the current flowing through a galvanometer, R the resistance of the galvanometer, together with any resistance which may be purposely connected in circuit with it, and let E be the e. m. f. which is sending current through the arrangement. Then $E = IR$ (from equation (218)). If the current be measured by means of the galvanometer, and R be known, E may be calculated. A galvanometer of high resistance is adapted to this use. Such a galvanometer is sometimes called a *potential* galvanometer. An *ammeter* may be used in the same way. When so used the instrument should have a high resistance. The ammeter scale can be made to give readings directly in volts. An ammeter so arranged is called a *voltmeter*. There is but one type of voltmeter, namely the *electrostatic voltmeter*, which is not essentially an ammeter.

424. Poggendorff's compensation method for measuring e. m. f. — Let AB (Fig. 222) be a portion of a circuit through which a current I is flowing. At two points C and D wires are connected leading through a sensitive galvanometer G, and a battery whose e. m. f. e is to be measured. The battery e is connected so as to tend to produce current in a direction opposite to the current which would flow through G if e were removed. The current I is then changed until the sensitive galvanometer G shows no deflection, after which adjustment the e. m. f. Ir between C and D is equal to the e. m. f. of the battery e; that is,

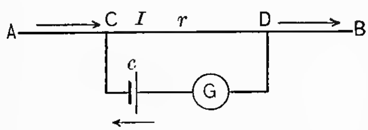

Fig. 222.

$$e = Ir. \tag{i}$$

If I be measured, absolutely, by means of a tangent galvanometer, or otherwise, and if r be known, this gives a fundamental determination of e. Let another battery, the e.m.f. of which is e', be put in place of e and the current through AB changed to the value I' until g again gives no deflection, then

$$e' = I'r. \tag{ii}$$

Dividing (i) by (ii), member by member, we have

$$\frac{e}{e'} = \frac{I}{I'}. \tag{iii}$$

The ratio $\frac{I}{I'}$ may be measured by means of a tangent galvanometer placed in the circuit AB. The ratio $\frac{e}{e'}$ is thus determined; and if e is known (*e.g.* if e is the e.m.f. of a standard cell, see article 406) e' may be calculated.

425. Measurement of the ratio of two e.m.f.'s by means of the slide-wire potentiometer. — A constant current is maintained in a stretched wire AC (Fig. 223) by means of a battery B. A battery of e.m.f. e and a sensitive galvanometer are connected to A, and to a sliding-contact d, which is moved until the galvanometer gives no deflection, then $Ir = e$, as in the previous paragraph. Another battery of e.m.f. e' is now put in place of e, and the contact d is again moved until g gives no deflection, then $Ir' = e'$. We have therefore

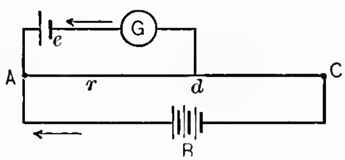

Fig. 223.

$$\frac{e}{e'} = \frac{r}{r'}. \tag{i}$$

Since the ratio $\frac{r}{r'}$ is equal to the ratio of the respective lengths of wire between Ad, and is thus easily determined, $\frac{e}{e'}$ is known.

Measurement of Magnetic Field.

426. Comparison of strengths of field at two places by the method of vibrations. — A magnet whose moment of inertia is K and magnetic moment is M, when suspended at a place at which the horizontal field is H, vibrates in such a way that

$$\frac{4\pi^2 K}{\tau^2} = MH, \tag{i}$$

in which τ is the period of the vibrations. See equation (197). If the magnet be suspended at a place at which the horizontal field is H', we have

$$\frac{4\pi^2 K}{\tau'^2} = MH'. \tag{ii}$$

Dividing (i) by (ii), member by member, we have

$$\frac{H}{H_1} = \frac{\tau_1^2}{\tau^2}, \tag{236}$$

so that if τ and τ_1 be observed, $\dfrac{H}{H_1}$ is known.

427. Comparison of strengths of field at two places by the method of deflections. — If a small magnet be suspended at one of the places and deflected through the angle ϕ by a large magnet, then

$$\tan\phi = \frac{h}{H}. \tag{i}$$

(See Art. 346, *second arrangement.*) If the small magnet be now suspended in another place where the field is H', and be again deflected through the angle ϕ' by means of the same large magnet at the same distance, then

$$\tan\phi' = \frac{h}{H'}. \tag{ii}$$

Dividing (i) by (ii), member by member, we have

$$\frac{H'}{H} = \frac{\tan\phi}{\tan\phi'}, \tag{iii}$$

so that the ratio $\frac{H'}{H}$ is known when ϕ and ϕ' have been observed.

428. Measurement of field by means of the tangent galvanometer. — Let a current I, measured, for example, by means of a silver voltameter, be sent through a tangent galvanometer, and the deflection ϕ observed; then

$$I = \frac{rH}{2\pi n} \tan \phi. \qquad (206\ bis)$$

If r and n are known, we may measure I and ϕ, and thus obtain data from which H may be calculated.

429. Measurement of strength of field by means of a suspended coil. — Let a current I, measured, for example, by means of a silver voltameter, be sent through a coil suspended as explained in article (361), then

$$T = \pi n r^2 I H \qquad (212\ bis)$$

is the torque acting upon the coil. If n and r are known and T be observed, H may be calculated.

430. Kohlrausch's method for the simultaneous measurement of the horizontal component H of the earth's magnetic field and of current. The coil of a tangent galvanometer is suspended so as to enable the measurement of the torque with which H acts upon it. This torque is

$$T = \pi n r^2 I H. \qquad (210\ bis)$$

At the same time the deflection ϕ of the needle of the galvanometer is observed. The value of the current is

$$I = \frac{rH}{2\pi n} \tan \phi. \qquad (206\ bis)$$

The quantities r and n being known, and T and ϕ being observed, these two equations enable the calculation of both C and H.

CHAPTER VII.

PRELIMINARY STATEMENTS CONCERNING ELECTROSTATICS.

431. Charging by contact and separation. Electric attraction and repulsion. — Many substances — for example, rosin and fur, — when separated, after having been brought into intimate contact by rubbing, are found to *attract* each other. In this condition the substances (both of them) are said to be *charged with electricity* or *electrified.* Two similar substances charged by similar treatment, for example two pieces of glass which have been rubbed with silk, *repel* each other. These forces of attraction and repulsion are ordinarily very weak.

Two kinds of charge. — Every charged body falls into one or the other of two classes, according as it is attracted or repelled by a given charged body. These classes refer to the *charge* on the body, not to the body itself; for a given body may be electrified so as to fall into either one of the classes. Charges which fall into the same class as that which is produced upon *glass which has been rubbed with silk* are called *vitreous* or *positive* charges. Charges like that which is produced when *rosin has been rubbed with fur or flannel* fall into the other class, and are called *resinous* or *negative* charges. Charges of the same kind repel each other; charges of unlike kinds attract each other.

432. Electroscopes. — On account of the smallness of the forces between charged bodies, some device must be employed to facilitate their detection. Sometimes one of the charged bodies is suspended by a silken thread, as in Fig. 224; sometimes a very small, light body is thus suspended and serves as an **electroscope.**

The materials usually selected are balls of pith or strips of gold leaf. In the former case, the instrument is called a pith-ball electroscope; in the latter, a gold-leaf electroscope (see Art. 435).

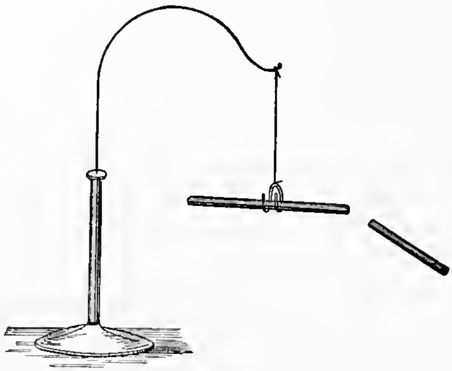

Fig. 224.

433. The pith-ball electroscope consists of a gilded ball of pith, suspended by a silk thread (Fig. 225). The ball is made of pith for the sake of lightness, and is gilded to make it a conductor. If a charged body is brought near such a ball,

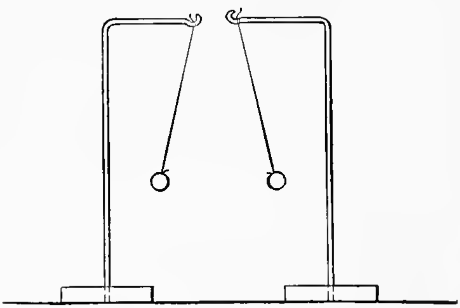

Fig. 225.

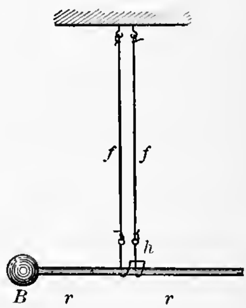

Fig. 226.

the ball is at first attracted (see Art. 435), and, upon touching the body, it is quickly repelled, showing that the ball has taken on a charge similar to that of the body. Two balls charged by contact with the same charged body repel each

other. If one ball has been charged by touching with glass which has been rubbed with silk, and the other by touching with rosin or hard rubber which has been rubbed with fur, the balls will attract each other. In this case, any charged body which attracts one of the balls is found to repel the other.

The following arrangement of the pith-ball electroscope is much more sensitive than the above. The pith ball *B* (Fig. 226) is attached to one end of a very light insulating rod *rr*, which is carried in a horizontal position in a light hook *h*, hung by two fibers *ff*.

434. Retention of charge. Electric conduction. — A charged body, however supported, loses its charge more or less rapidly. A gilded pith ball loses its charge almost instantly if it is touched with a metal rod held in the hand, or with a wetted string or stick: it loses its charge much more slowly if touched with a wand of dry wood, and very slowly indeed if touched with, or supported by, a rod of glass or hard rubber, or a dry silk thread. In these cases, the charge is thought of as passing along the wire or rod to the hand, and thence to the earth. This action is called *electric conduction*. A substance along which a charge can pass is called a *conductor*. Metals and wet substances are good conductors, dry wood is a poor conductor; hard rubber, glass, and air are very poor conductors, or *insulators*. A charged body is said to be *insulated* when it touches nothing but insulators.

The reason for thinking that a charge passes along a wire or other conductor is, that if a charged body be connected by means of a wire with an uncharged insulated body, the second body becomes charged.

A charged insulator — for example, a charged rod or sheet of glass or hard rubber, — however supported, does not give up its charge quickly, for the portions of the body which are remote from the points of support are insulated by the intervening portions of the body itself. A charged insulator must be left to

itself for a long time, or literally bathed in a conductor to be completely discharged. For example, a charged glass rod may be quickly discharged by passing it through a flame, because hot gases are good conductors. A charged insulator is effectively discharged by sweeping over it a set of sharp metal points, as exemplified in the use of a *collecting comb* on an electric machine.

435. Charging by influence; the two fluid theory. — In order to describe and to fix in mind the phenomenon of *charging by influence*, it is convenient to think of the two kinds of electric charge as *fluids*. When these two fluids are in a body in equal amounts they neutralize each other, and the body has no charge. One fluid is in excess in positively electrified bodies, and the other fluid is in excess in negatively electrified bodies. It is further convenient to think of the attraction or repulsion between charged bodies as acting between these *fluids*. A much more complete theory of electrical action is given in Chapter VIII.

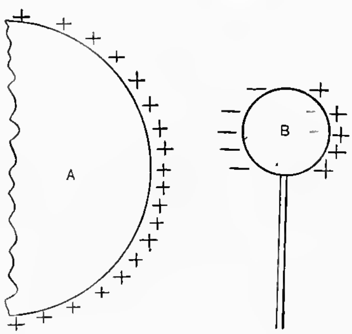

Fig. 227.

If a neutral, conducting body B (Fig. 227) is brought near to a (positively) charged body A, the two electric fluids in B are to an extent separated by the action of the charge on A. The negative fluid is attracted to the side of B adjacent to A, and the positive fluid is repelled to the remote side of B. In this condition B is, on the whole, attracted by A, for the negative charge on B is nearer to A than the positive charge on B. The two opposite charges thus produced on the body B are *equal* numerically, for if B is moved to a distance from A it will be found to be neutral. If B, while it is near A, is touched with a conductor held in the hand, the positive fluid

will flow off, leaving the negative fluid *bound* on B by the attraction of the positive charge on A. If B is now removed to a distance from A, this bound charge will be distributed over B, and B will exhibit all the characteristics of an ordinarily charged body.

436. The gold-leaf electroscope consists of a metal disk or ball D and rod R (Fig. 228), from the lower end of which two gold leaves are hung side by side. The whole is supported in a glass case *cc*, which protects the gold leaves from air currents. The sides of *cc* are lined with metal foil to increase the sensitiveness and to prevent the formation of troublesome charges on the inside surface of the glass. If the disk is charged, the charge will spread over the rod and leaves, and the two leaves becoming similarly charged will repel each other and diverge, thus indicating the charge. Suppose the rod and disk to be charged *positively;* then a *negatively* charged body held near the disk will draw a larger part of this *positive* charge into the disk, leaving less charge on the leaves, which will become less divergent. If the *negatively* charged body is moved nearer to the disk, all the *positive* charge may be drawn into the disk, leaving no charge upon the leaves, which then hang vertically. If the *negatively* charged body is moved still nearer to the disk, the *positive* charge drawn into the disk may leave a *negative* charge on the leaves, which again diverge. A *positively* charged body held near the disk pushes the positive charge downward from the disk and rod more and more as the body is brought nearer and nearer to the disk, causing the leaves to diverge more and more. The gold-leaf electroscope is much more sensitive than the pith-ball electroscope. A vastly more sensitive instrument than either (quadrant electrometer) is described in Art. 483.

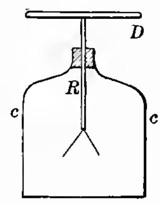

Fig. 228.

437. Transfer of the entire charge from one body to another.— If a charged conducting body A (supported by an insulating handle) be brought into contact with an uncharged conductor, the charge will be divided between them. *If the charged conductor A is introduced, through a small hole, and touches the inside of a hollow insulated conductor, it gives up all its charge. This is the case even though the hollow conductor be already highly charged.* This is shown by the fact that the body A is found to be entirely without charge when removed from the hollow conductor and tested.

438. The electrophorus. — This is a device for the production of a charge by influence. It consists of a plate D (Fig. 229) of rosin or hard rubber, which has been electrified (negatively) by beating it with a piece of fur or flannel, and a disk M of metal provided with an insulating handle H. When the metal disk is brought near to the negatively charged plate of rosin and touched with the finger, it is left with a *bound* charge of positive electricity. This bound charge remains on M as a free charge when M is removed to a distance from D (Art. 435). This operation may be repeated indefinitely.

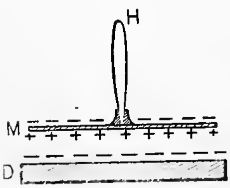

Fig. 229.

With the electrophorus only small quantities of electric charge can be produced. It is possible, however, to devise apparatus for the *continuous generation* of charge. Such a device is called an **electrical machine.** There are two types: (a) frictional machines, in which the method of contact and separation is employed; (b) influence machines, in which the operation is essentially that of the electrophorus.

439. The frictional electric machine. — This machine, in its most approved form, consists of a rotating glass disk DD (Fig. 230), the various parts of which come in succession into

intimate contact with two leather cushions AA, smeared with an amalgam of tin, zinc, and mercury. The surface of the glass plate as it leaves these cushions is left highly charged with positive electricity, while the cushions are left negatively electrified. The negative charge flows into the insulated conductor N, which is connected with the cushions by means of

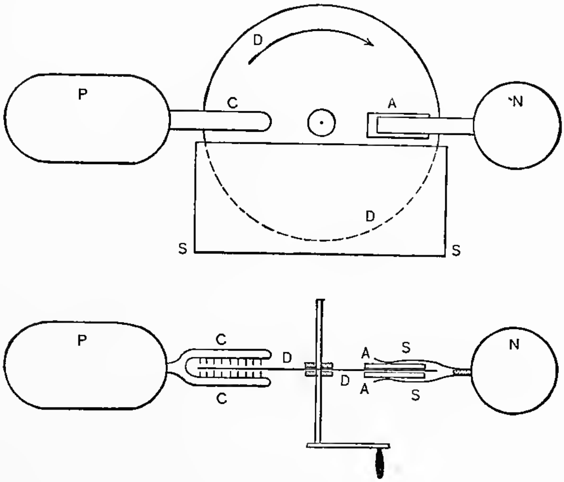

Fig. 230.

the springs SS. The positive charge on the glass plate is collected by the sharp points of the metal *combs cc*. The silk apron *s*, *s*, between the folds of which the surfaces of the plate pass, enhances the action of the machine.

440. Influence electric machines. — The electrophorus is the simplest arrangement for the generation of charge by influence. As has already been stated, the various influence machines are essentially similar to it in action, except that the *inducing* charge is generated by the machine also. The *revolving doubler* is the oldest form. The Holtz machines, next in order, were modified by Töpler, and the result is the *Töpler-Holtz machine*, now extensively used. The *Wimshurst machine*, which is perhaps the simplest of all, is also much used.

441. The revolving doubler. — Two plates of metal, *CD*, Fig. 231, called *carriers*, are mounted upon insulating supports and rotated so as to pass along the dotted line *ll*. In the position shown in the figure, *C* and *D* are touched momentarily by the metal brushes 2 and 4, which are fixed on the ends of a stationary metal "neutralizing rod." Under the influence of the charged conductors *A* and *B*, these carriers take on positive and negative charges, as shown. The rotation carries *C* into the interior of the conductor *A*, where it is momentarily touched by the metal brush 1, giving up its entire charge to *A*. The carrier *D* at the same time gives up its charge to *B* in the same manner. As the carriers pass out from *A* and *B* they are again touched by the brushes 2 and 4, taking on fresh charges, which are given up to *A* and *B* as before, and so forth. The infinitesimal charges imparted to *C* and *D* by the mere contact of the brushes *C* and *D*, when *A* and *B* are discharged, are sufficient, because of the multiplying action of the machine, to bring the arrangement very quickly into active operation. This arrangement is for this reason said to be *self-exciting*.

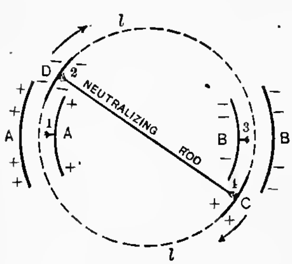

Fig. 231.

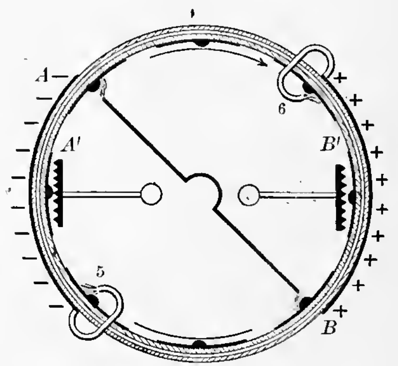

Fig. 232.

442. The Töpler-Holtz machine. — The action of the *Töpler-Holtz* machine is identical to that of the revolving doubler except that *the charged bodies A and B* (Fig. 231) *each consists of two parts, AA' and BB'* (Fig. 232). *The charge which is*

drawn from the machine is taken from one A' and B' of these parts, leaving the charge on the other part, so that the machine may continue in active operation. The machine consists of a circular disk of varnished glass, upon which the six to ten or twelve carriers are fixed. These carriers are provided with metal buttons which are touched momentarily by the various metal brushes as the glass disk revolves. Behind this rotating disk is a fixed disk, upon which are strips of tin foil or paper (inductors) which carry the inducing charges. The parts which

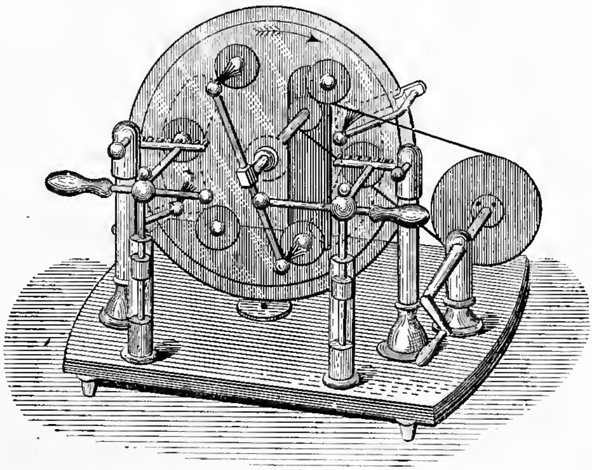

Fig. 233.

collect the charge for use are in front of the rotating disk. The essential features of the machine are best represented by a diagram of the kind shown in Fig. 232, where the four metal carriers and the inductors A and B are arranged on glass cylinders.

Just before the charged carriers pass between A and A' and B and B', where they give up all their charge to A' and B', they touch the brushes 5 and 6 (Fig. 232), and give part of their charge to the inductors A and B.

The Töpler-Holtz machine is self-exciting. It is not so much

influenced by moisture as is the frictional machine. A view of this machine is shown in perspective in Fig. 233.

443. The Wimshurst machine. *Preliminary.* — Let P (Fig. 234) be a metal point, connected to earth, near a charged surface AB. Let CD be a sheet of glass. The lines of electric stress from the charge AB converge upon the point P, being very little disturbed by the presence of the glass sheet CD. The electric field in the neighborhood of P is thus intense enough, if AB is at all strongly charged, to break down the dielectric; viz., the air, between the point and CD. Then the state of affairs is that shown in Fig. 235, in which only those lines are shown which have broken down between P and CD. The small portion of the surface of CD which faces the point is thus negatively charged, and the *amount of charge on this small portion is equal to the amount of positive charge on* the much larger part of AB, from which the lines emanate which have been broken down between P and CD. If the plate CD is moved to the left, fresh lines of electric stress will crowd between the point and CD, and by their continual breaking down, the surface of CD as it moves out from under P will be left *much more strongly charged* than the plate AB. This plate AB may itself be charged by moving it to the right under a point P' under the inducing action of CD, as

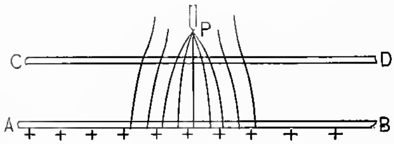

Fig. 234.

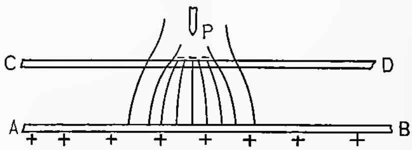

Fig. 235.

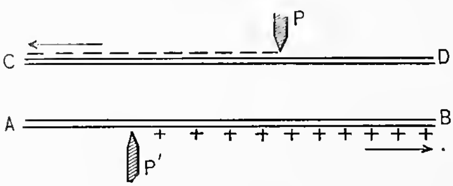

Fig. 236.

shown in Fig. 236. The charges on AB and CD will thus grow more and more intense until checked by the rapidly increasing leakage from the surface of the plates. The negative charge on CD after it has passed well beyond the point P',

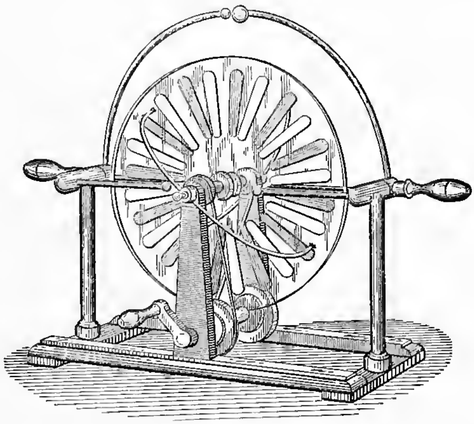

Fig. 237.

and the positive charge on AB after it has passed well beyond the point P, may be collected by metal combs and used for any purpose.

Many machines of the Wimshurst type have metallic carriers upon the plates, and metal brushes in place of the points PP' (Fig. 236).

With such an arrangement the charge necessary to start the action is infinitesimal, and the self-exciting property of the machine is improved.

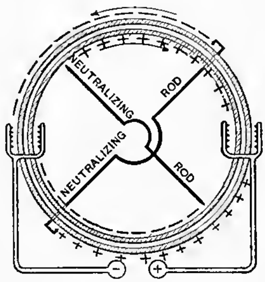

Fig. 238.

The carriers on AB have a multiplying action upon the uncharged carriers on CD. The performance of the machine is not essentially changed by the use of carriers. The use of carriers has the disadvantage that it increases the leakage and thus prevents the arrangement from attaining to such

intense action as is possible with plain glass plates. The Wimshurst machine as designed for practical use is shown in perspective in Fig. 237, which illustrates a self-exciting machine with metal carriers. The essential features of this machine are represented in Fig. 238, in which the glass disks are represented as hollow cylinders for the sake of clearness. The machine represented in Fig. 238 is one without metal carriers.

Remark. — Articles 484 and 485 are essential to the full understanding of the action of the Wimshurst machine.

444. Concentrated charge. Distributed charge. — When the dimensions of a charged body are small compared to the distances from that body to other bodies upon which the charge acts, the dimensions of the charged body may be ignored, and the charge may be thought of as *concentrated* at a point. When this is not the case the charge is said to be a *distributed* charge.

All preliminary definitions and developments apply to ideal concentrated charges. Conceptions of what takes place in case of distributed charges are then arrived at by superposition. (See Art. 450.)

445. Coulomb's law. Strength of charge. — *The force with which two concentrated charges attract (or repel) each other is inversely proportional to the square of the distance between them.* That is,

$$F = \frac{c}{d^2}, \tag{i}$$

in which c is a quantity which has a determinate value for every pair of charged bodies. The value of c depends, however, upon the medium (*dielectric*) in which the charged bodies are placed, *e.g.* whether in vacuum, air, CO_2, oil, or other dielectric.

Taking the values (*in air*) of c, equation (i), for each of the three pairs of three given charges, we may find three quantities Q_1, Q_2, and Q_3 associated singly with the three charges, exactly as in Arts. 330 and 331, such that

$$F = \frac{Q_1 Q_2}{d^2}, \tag{237}$$

in which F is the force with which two charges Q_1 and Q_2 repel (or attract) each other at distance d from each other in air. The quantities Q_1 and Q_2 are called the *strengths* of the charges, or the *quantities* of electricity upon the respective charged bodies.

For any dielectric other than air equation 237 becomes

$$\left.\begin{aligned} F &= K\frac{Q_1 Q_2}{d^2} \\ \text{or} \qquad F &= \frac{1}{K}\frac{Q_1 Q_2}{d^2} \end{aligned}\right\} \tag{238}$$

in which $K\left(\text{or } \frac{1}{K}\right)$ is a determinate constant for the given dielectric. This quantity K or $\frac{1}{K}$ is called the *specific inductive capacity* or sometimes simply the *constant* of the given dielectric. Another definition (not independent) of specific inductive capacity is given in Art. 481. See Chapter IX for further discussion of electric charge and specific inductive capacity.

The force F, in equation (1) is considered *positive* when it is a repulsion, and negative when it is an attraction. Therefore the product $Q_1 Q_2$ is positive when Q_1 and Q_2 are charges of the same kind, and negative when Q_1 and Q_2 are dissimilar charges. (See Art. 471.)

446. Unit charge. — In conformity with equation (237) a *unit charge*, or unit *quantity* of electricity, is defined as a charge, which at a distance of one centimeter from an equal charge will exert upon it a force of one dyne. This is not at all the same

unit as the *coulomb* already defined. The unit here defined is called the "electrostatic" unit of charge. 3.00×10^9 of these units are equal to one coulomb.

Volume density of electric charge. — Let ΔQ be the electric charge in a small region of volume $\Delta\tau$, then the ratio $\frac{\Delta Q}{\Delta\tau}$ is called the *volume density* of electric charge in that region.

CHAPTER VIII.

PROPERTIES OF THE ELECTRIC FIELD.

447. Electric field; intensity at a point. — An electric field is a region in which an electric charge is acted upon by a force pulling it in some direction or other. For example, the neighborhood of a charged body is an electric field, for another charged body in that neighborhood is acted upon by a force. *The force which acts upon a small charged body when placed at a given point in an electric field is proportional to the charge on the body.* The proportionality factor is called the *intensity* or *strength* of the field at the given point. That is,

$$F = Qf, \tag{239}$$

in which F is the force acting upon a charge Q, placed at a point in an electric field where the intensity of the field is f. The *intensity*, at a point, of an electric field is a vector quantity (a distributed vector), and its direction is understood to be that of the force which acts upon a *positive* charge. The force which acts upon a negative charge is in the opposite direction. The intensity of an electric field f at a point may be regarded as a stress* in a medium. The corresponding strain is called *electric displacement*. The medium which sustains this stress and strain is called the *dielectric*.

448. Unit of field intensity. — In conformity with equation (239) an electric field is considered to be of unit intensity when it acts upon a unit charge with a force of one dyne. This is called the "*electrostatic*" unit of field, and is equal to 300 volts per centimeter.[2] (See Art. 358.)

* Electric stress is not *force per unit area*, but, owing to the convention in its measurement involved in equation (239), it is $\sqrt{\text{force per unit area}}$. (See Art. 122, Vol. I.)

449. Intensity of the electric field at a point distant d from a concentrated charge Q. — Equation (237) may be written $F = Q_1\left(\frac{Q_2}{d^2}\right)$. Comparing this with equation (239), we see that $\frac{Q_2}{d^2}$ is the intensity of field in the neighborhood of the charge Q_1, due to the charge Q_2. We have, therefore,

$$f = \frac{Q}{d^2}, \tag{240}$$

in which f is the intensity of the electric field at a point distant d from a concentrated charge Q. The lines of the field f are directed *away* from Q when Q is positive, and towards Q when Q is negative.

450. Superposition. — Any two causes which, acting singly, produce at a point p (Fig. 239) electric intensities represented by the lines 1 and 2 respectively, produce, when acting together, an intensity at p represented by the line 3, which is the vector sum of 1 and 2.

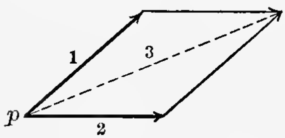

Fig. 239.

Example. — The field at p (Fig. 240), due to two concentrated charges, $-Q_1$ and $+Q_2$, is found by taking the vector sum of the two intensities 1 and 2 due to Q_1 and Q_2 respectively.

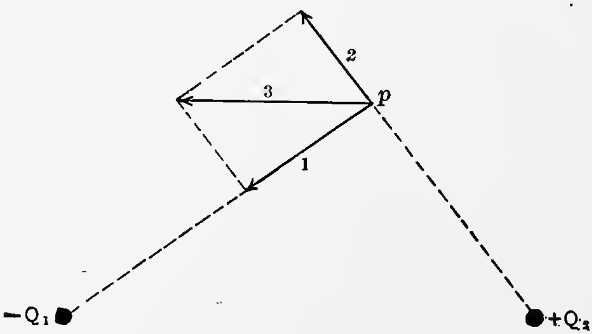

Fig. 240.

451. Lines of force or lines of stress in an electric field. — A line drawn through an electric field, so as to be at each point in the direction of the field at that point, is called a *line of force* or a *line of stress*. Lines of force in the electric field serve purposes precisely analogous to the lines of force of the magnetic field (see Arts. 348 and 349). For examples, see Arts. 461, 464, etc.

452. Electric flux. — Consider a small plane area ΔS at a point in an electric field, at which point the electric intensity is f (Fig. 241). Let e be the angle between f and the normal to ΔS. Then $f \cos e$ is the component of f perpendicular to ΔS. *The product of this normal component of f into the area ΔS is called the electric flux across the area.*

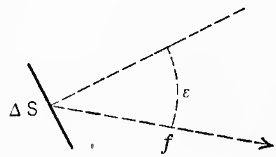

Fig. 241.

The electric flux across any extended surface is

$$I = \sum f \cos e \Delta S. \tag{241}$$

That is, the extended surface is broken up into elements ΔS. Each of these elements is multiplied by the normal component of f at the element, and the sum of these products is taken. (See Art. 320.)

Unit flux may be defined (equation (241)) as that flux which crosses one square centimeter of a surface normal to a field of unit intensity.

Tube of flux; unit tube. — Imagine lines of force to be drawn through each point of the periphery of any small loop in an electric field. These lines of force form a tubular surface called a tube of flux. If the electric flux across a diaphragm to this tube is unity, the tube is called a unit tube. (See Art. 325.) A tube of electric flux is sometimes called a *Faraday tube*.

453. Gauss's theorem. — *The electric flux outwards through any closed surface is equal to 4π times the total electric charge inside the surface.* (Compare Art. 324.)

This fact may be expressed by means of the equation

$$I = 4\pi Q. \tag{242}$$

The following is a proof of Gauss's theorem in three steps:

(1) *Case of a concentrated charge situated at the center of a spherical surface.*

Let Q be a concentrated charge. Describe a sphere of radius r with its center at Q. The field intensity at the surface of this sphere is $f = \frac{Q}{r^2}$ from equation (240), and normal to the spherical surface. The area of the sphere is $4\pi r^2$, and the electric flux out of the sphere is the product of the field intensity into the area of the sphere; that is, $I = 4\pi r^2 \times \frac{Q}{r^2} = 4\pi Q$. Q.E.D.

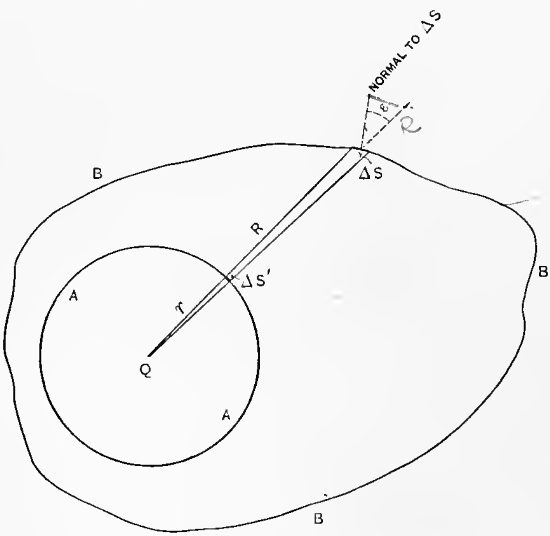

Fig. 242.

(2) *Case of a concentrated charge within any closed surface.*

Let Q, Fig. 242, be a concentrated charge. Let A be a spherical surface of radius r, center at Q, and B any given sur-

face enclosing Q. Let the surfaces A and B be broken up into *corresponding elements by acute cones with their apices at* Q. Consider a given pair $\Delta S'$ and ΔS of these surface elements. Let R be the distance of ΔS from Q, and ϵ the angle between R and the normal to ΔS. The field intensity at ΔS is $\frac{Q}{R^2}$. It is parallel to R so that $\frac{Q}{R^2}\cdot\cos\epsilon\cdot\Delta S$ is the flux across ΔS. Now $\cos\epsilon\cdot\Delta S$ is the projection of ΔS upon a plane parallel to $\Delta S'$, *i.e.* perpendicular to R, so that

$$\frac{\cos\epsilon\cdot\Delta S}{\Delta S'}=\frac{R^2}{r^2}, \text{ or } \cos\epsilon\cdot\Delta S=\frac{R^2}{r^2}\Delta S'.$$

The flux across ΔS, viz., $\frac{Q}{R^2}\cos\epsilon\cdot\Delta S$, is, therefore, equal to $\frac{Q}{R^2}\cdot\frac{R^2}{r^2}\Delta S'$ or to $\frac{Q}{r^2}\Delta S'$, but this is the flux across $\Delta S'$. Therefore *the flux is the same across any two corresponding elements of A and B.* Each element of A corresponds to some one* element of B and *vice versa*, so that the total flux across the given closed surface B is equal to the total flux across the spherical surface A, *i.e.* it is equal to $4\pi Q$. Q.E.D.

(3) Consider a surface enclosing any number of concentrated charges Q (a distributed charge). The outward flux across the enclosing surface due to each charge Q is $4\pi Q$, and, by the principle of superposition (Art. 450), the outward flux due to all the charges is 4π times the sum total of the charges inside the surface. Q.E.D.

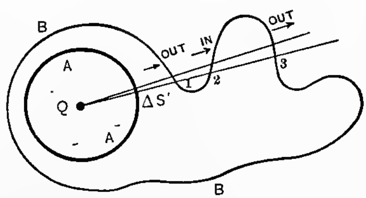

Fig. 243.

*The case exhibited in Fig. 243 does not conform strictly to the above statement, but in such case there must be an *odd* number of elements of B corresponding to a given element $\Delta S'$ of A. The above discussion shows that the flux across each of these corresponding elements is the same as across $\Delta S'$. Further, the flux must be alternately *outwards* and *inwards* across the elements 1, 2, 3, etc., therefore only the last element contributes effectively to the outward flux.

Remark. — Gauss's Theorem is essentially the same thing as the theorem, Art. 322, referring to the relation between surface and volume integrals. That theorem, together with the proof here given, shows conclusively that *volume density* of electric charge and *divergence* of electric field are identical, if, indeed, this requires other proof than is at once evident from equation (240). The fact that charge resides on the surface of charged conductors does not render the conception of volume integral, of volume density, of electric charge meaningless, for charge must merely reside in a very thin layer of the surface of a charged conductor where the volume density is very great, but by no means infinite.

454. Electric charge resides wholly on the outside surface of charged conductors. — If a metal ball with an insulated handle be introduced through a small opening into the interior of a hollow charged conductor, it is found never to receive any charge no matter how highly charged the hollow conductor may be. This statement, which has already been discussed in Art. 437, holds true, however thin (practically) the hollow vessel or however large the ball, provided only that the opening is small, or covered with a lid while the ball is inside. While the ball is in contact with the inside of the hollow conductor, it is essentially a part of it, and if any of the charge on the hollow conductor were on its inner surface or distributed through its material, some charge would necessarily be given to the ball. Therefore the charge on a conductor resides on its outside surface.

Surface density of electric charge. Let ΔQ be the charge upon a small portion of area ΔS of the surface of a charged conductor. The ratio $\frac{\Delta Q}{\Delta S}$ is called the surface density of the charge upon the body at the element ΔS. The variation of surface density of charge is easily shown by means of a proof-plane.* This is found to be more heavily charged when touched

* The name given to a small conducting disk carried on the end of an insulating rod.

to the projecting portions of a charged conductor than when it is touched to the portions which are more nearly flat.

455. A further experiment with hollow conductors.— Let a hollow conductor *CC*, such as a tin can, be set upon the disk of a gold-leaf electroscope, as shown in Fig. 244. If a (positively) charged body *B* is introduced into *CC*, the leaves

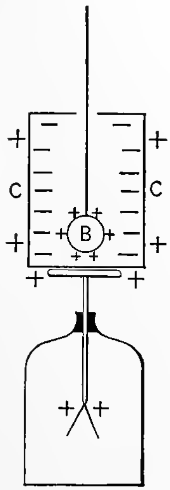

Fig. 244.

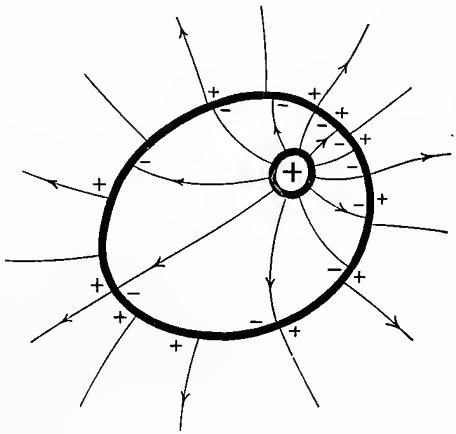

Fig. 245.—Distribution of outside charge independent of position of charged body inside.

of the electroscope will diverge, because the + charge on *B* induces a − charge on the inside of *CC*, and an equal (see Art. 456) + charge on the outside of *CC*.

The divergence of the electroscope leaves is not in the least changed by moving B about inside of CC, or even by bringing B into contact with the inside of CC.

If two equally and oppositely charged bodies are introduced into CC, the electroscope gives no indication whatever.

These facts indicate that a charge $+Q$ on the body *B*, placed inside of *CC*, induces an equal and opposite charge $-Q$

on the inside, and a charge $+Q$ on the outside of CC; and that the distribution of the charge $+Q$ on the outside of CC is independent of the position of B inside of CC. (See Art. 460.) When B is brought into contact with the inside of CC, the positive charge on B and the equal negative charge on the inside of CC neutralize each other, leaving the positive charge on the outside of CC undisturbed. Figure 245 shows the state of affairs when a charged body is placed inside of a hollow conductor, and Fig. 246 shows the state of affairs when two equally and oppositely charged bodies are placed inside a hollow conductor.

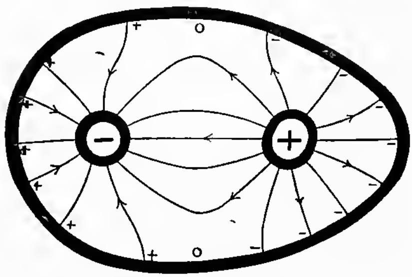

Fig. 246. — No outside charge.

456. The production of positive electricity is always accompanied by the production of an equal quantity of negative electricity. — If an electric machine in operation, or any arrangement whatever which generates electric charge, is caged in a metal box which is connected with a gold-leaf electroscope, it is found that the outside of the box *shows no trace of charge*. This shows that *equal quantities of positive and negative electricity are always generated.*

457. Character of the field in the neighborhood of an isolated charged sphere. — The sphere being isolated (*i.e.* at a distance from all other bodies), the charge Q on the sphere is uniformly distributed over the sphere, and the electric field must be everywhere directed away from or towards the sphere, according as its charge is positive or negative. This field must be of the same intensity at all points equidistant from the sphere. Describe a spherical surface of radius r concentric with, but

larger than, the charged sphere. Let f be the field intensity at the surface of this sphere of radius r and area $4\pi r^2$. Then $4\pi r^2 f$ is the electric flux across this spherical surface, which must be equal to $4\pi Q$ (from Art. 453). Therefore $r^2 f = Q$, or $f = \frac{Q}{r^2}$. This is identical to equation (240). Therefore *the charge on an isolated (uniformly charged) sphere acts on external points as if it were concentrated at the center of the sphere.*

The electric field inside a uniformly charged spherical shell is known to be zero from the experiments described in Arts. 437 and 454. It can likewise be shown to be zero from Gauss's theorem, as follows: The electric field f, inside the sphere, at a distance r from its center, must be *radial* from symmetry. Therefore the flux across a sphere of radius r is $4\pi r^2 f$, which is equal to $4\pi \times$ charge inside. But there is no charge inside; therefore f is zero.

458. Electric field in the neighborhood of an infinite uniform plane layer of electric charge. — Let σ be the electric charge per unit area of the layer. Then $a \cdot \sigma$ is the charge of area a of the layer, and $4\pi a\sigma$ is the total outward flux from this charge (Art. 453). Now the electric field f in the neighborhood of the layer is, by symmetry, perpendicular to the layer and of the same value on the two sides. Therefore the outward flux af from area a of the layer on one side is equal to $\frac{1}{2} \times 4\pi a\sigma$, so that

$$af = 2\pi a\sigma$$

or

$$f = 2\pi\sigma. \tag{243}$$

Remark. — This expression holds true for a point very near to any finite, but infinitely thin, charged layer.

459. Electric field in the neighborhood of a uniformly charged cylindrical rod of infinite length. — Let f be the field intensity at distance r from the axis of the rod; f is, by symmetry, perpendicular to the rod. The outward flux from length l of the

rod is $2\pi r \cdot l \cdot f$, which is equal to 4π times the charge lv on that portion of the rod; therefore

$$f = \frac{2v}{r}, \tag{244}$$

in which v is the charge per unit length of the rod.

The electric field inside a uniformly charged cylindrical shell can be proven to be zero from Gauss's theorem in a manner similar to the proof of the corresponding proposition for a uniformly charged sphere. (See Art. 457.)

460. Screening action of conductors; definition of a conductor. — Consider two regions which are entirely separated by sheet metal; for example, the regions on the two sides of an indefinite plane sheet of metal, or the regions outside and inside of a closed metal box. It is found that *in no case do stationary electric charges in one region have any influence in the other region.* This fact seems to indicate that the ether in conducting substances cannot sustain the stress which constitutes electric field. Indeed, we may define a *conductor as a substance in which the ether cannot sustain electric stress.*

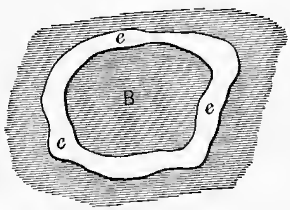

Fig. 247.

Thus if a solid body B (Fig. 247) be entirely separated from surrounding solids by an empty space e, then a distortion of the surrounding solid cannot affect B, nor can a distortion of B affect the surrounding solid.

461. Electric charge may be regarded merely as the ending of lines of electric stress.* — These lines of electric stress always

* These lines of stress may end *on the surface* of conductors, or *on the surface* or *in the interior* of insulators. At the surface between two dielectrics, *e.g.* glass and air, the electric stress may be continuous while the electric strain or displacement is *discontinuous.* This discontinuity of electric strain is also electric charge. In the text, charges on the surface of conductors only are discussed for the sake of brevity and simplicity.

end on the *surface* of a conductor, that is, they do not penetrate into the conductor, for electric stress cannot be sustained inside of a conducting substance. Article 462 will clear up this relation between lines of stress, or rather Faraday tubes, and electric charge. The following analogy (by no means complete) may be found helpful.

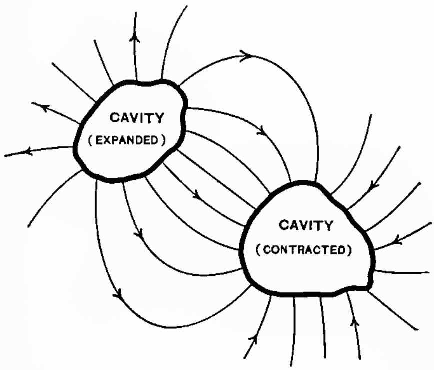

Fig. 248.

A solid body is ordinarily put under stress by forces applied at its surface. A case analogous to the production of charge upon an insulated conductor would be *the production of stress in an extended solid by acting on it from the inside of a cavity.* Fields of stress would in such a case begin and end at the surface of such cavities. Figure 248 represents the lines of stress in an extended solid in the neighborhood of two cavities. The student of electrostatics will find it advantageous to look upon conductors as cavities (*i.e.* as vacuity) in the ether!

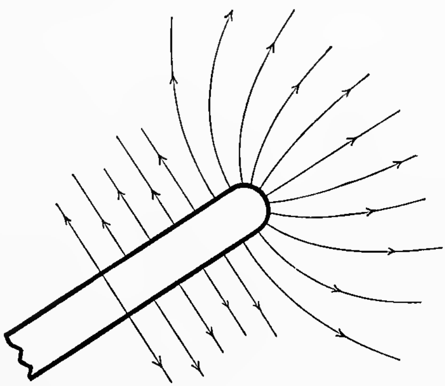

Fig. 249.

Representation of charge.—From the above it is clear that the most essential method of representing electric charges upon bodies is by drawing lines of stress *out from* positively charged bodies and *in towards* negatively charged bodies. Figure 249 shows a typical case.

462. Coulomb's theorem. — The field intensity f, at a point near the surface of a charged conductor, is equal to 4π times the surface density σ of the charge on the surface near that point. That is,

$$f = 4\pi\sigma. \tag{245}$$

Proof. — Consider a small portion of area a of the charged surface of the conductor. The charge on this portion is $a\sigma$, and the outward flux from it is $4\pi a\sigma$ (Art. 453). Now, since the field is zero in the interior of the conductor, this flux is all towards the outside. Let f be the intensity of the electric field very near to the charged surface. Describe a surface parallel to and very near the charged surface. The outward flux from the portion a of the charged surface *crosses* area a of this parallel surface, and, since f is perpendicular to the surface, this flux is equal to af, so that $af = 4\pi a\sigma$, or $f = 4\pi\sigma$. Q.E.D.

From this theorem it follows that where a *unit tube of electric flux or a Faraday tube starts* from a charged surface, *there* are 4π units of positive electricity, and where it *ends*, *there* are 4π units of negative electricity.

463. Outward pull upon the surface of a charged conductor. — Consider a small element, of area a, of the surface of a charged conductor. Since the charge is all on the surface, this small element is practically a finite, but infinitely thin uniform layer of charge, so that the electric field on both sides very near it is $f = 2\pi\sigma$. This is the field *due* to the charged element. (See Art. 458.) However, as a matter of fact, the field on the inside of this element is zero and on the outside of it is $4\pi\sigma$. (See Arts. 437, 454, 462.) Therefore, superposed upon the field due to the element *is an outward field in the region of the element equal to* $2\pi\sigma$, *which is due to all the remaining charge upon the charged body.* The charge on the element is $a\sigma$, and the force pulling this element outwards is (by equation (239)) $F = a\sigma \cdot f$ or $F = a\sigma \times 2\pi\sigma$; where $f (= 2\pi\sigma)$ is the

field at the element due to the remainder of the charge upon the conductor. Dividing by the area of the element, we have

$$F = 2\pi\sigma^2, \tag{246}$$

in which F is the outward *pull per square centimeter* on the charged surface. Substituting for σ its value in terms of the intensity of the electric field near the charged surface, from equation (245) we have

$$F = \frac{1}{8\pi} f^2. \tag{247}$$

That is, the force per unit area with which the dielectric (air) surrounding a charged conductor pulls on the surface of the charged conductor is equal at each point to $\frac{1}{8\pi}\times$ the square of the intensity f of the electric stress at that point.

The dielectric is *everywhere* under tension in an electric field, and the pull per unit area in the direction of the field at a point is equal to $\frac{1}{8\pi}f^2$. It can be shown that equilibrium of the dielectric requires two equal *pushes* $\left(\frac{1}{8\pi}f^2\right)$ to act at right angles to f, and that any small cubical portion of the dielectric having one pair of faces perpendicular to f is acted upon by a *pull* of $\frac{1}{8\pi}f^2 \frac{\text{dynes}}{\text{q. c. m.}}$ across these faces, and by equal *pushes* across the remaining faces.

The lines of force in an electric field may then be thought of as "tending to shorten themselves and as repelling each other sidewise."

464. Lines of force perpendicular to the surface of a charged conductor. — Let AB, Fig. 250, represent the surface of a conductor, and let the electric field near AB in the dielectric be *parallel* to AB. The Faraday tubes in the dielectric will push on each other sidewise, and there being no Faraday tubes

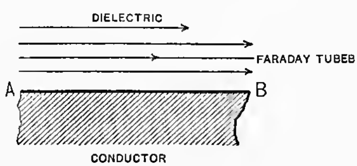

Fig. 250.

in the conductor (at least after a permanent state has been reached), there cannot be equilibrium. That is, we may think of the more remote Faraday tubes as pushing those adjacent to the surface *into* the conductor where they break down. Therefore an established electric field can have *no component*, near to a conducting surface, parallel to that surface, so that the electric field at the surface of a conductor is perpendicular thereto.

465. Explanation of charging by influence. — Let *A*, Figs. 251 and 252, be a positively charged body, from which Faraday

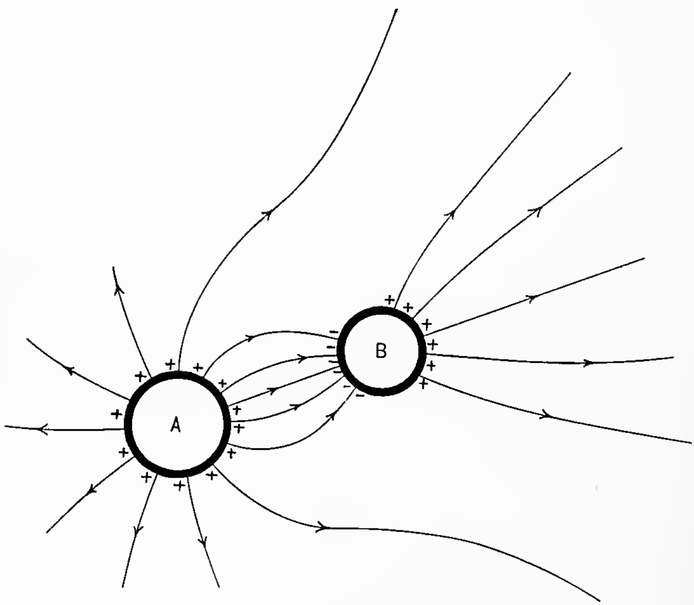

Fig. 251.

tubes pass out in all directions. If a conducting body *B* be brought into this field, the Faraday tubes will be pushed sidewise into this body *B*, until the field takes the form shown in Fig. 251. The side of *B* adjacent to *A* will thus be charged negatively and the opposite side positively. If now a conductor

be connected to B (no matter what its direction, so that it connects to earth), the Faraday tubes will be pushed sidewise into it and into the ball until the field assumes the character indi-

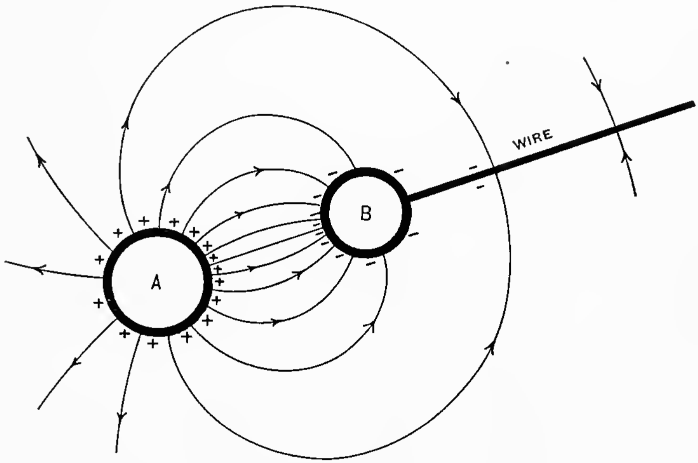

Fig. 252.

cated in Fig. 252. The body B now has only negative charge, which will be left as a free charge if B is insulated and removed to a distance from A.

466. Work done in moving an electric charge in an electric field. — Let a charge Q be moved from p to p', Fig. 253, along a given path, through an electric field. Let the charged body be at a given element Δs of the path. Let f be the intensity of the electric field at this element, and let ϵ be the angle between f and Δs. Then Qf is the force in the direction of f which acts upon the charge Q, and $Qf \cdot \cos \epsilon$ is the resolved part of this force in the direction of Δs, so that $\Delta W = Qf \cdot \cos \epsilon \cdot \Delta s$ is the work done by the force Qf, as Q is

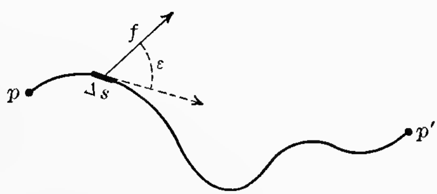

Fig. 253.

moved along Δs. Summing up such small amounts of work for all the elements of the path, we have

$$W = \sum Qf \cdot \cos \epsilon \cdot \Delta s$$

or

$$W = Q \sum f \cdot \cos \epsilon \cdot \Delta s \tag{248}$$

in which W is the work done upon Q by the electrical forces as it is moved from p to p'. To move Q from p' to p, an equal amount of work must be done by some outside agent.

467. Proposition. The work done as Q is moved from p to p' is independent * of the path over which it is carried. — *Proof.* Consider two paths from p to p', for which the work done is *not* the same. Let Q pass from p to p' over the path which furnishes the *greater amount* of work and back from p' to p over the other path. The whole system will then be in exactly its initial state,* while work will have been gained. This is contrary to the principle of the conservation of energy; therefore there can be no two paths for which the work is different.

The work done in carrying Q from p to p', being independent of path, is dependent only upon the state of the electric field, and the positions of p and p'. This work is *proportional* to Q (equation 248). That equation may therefore be written:

$$W = QE, \tag{249}$$

in which

$$E = \sum f \cdot \cos e \cdot \Delta s. \tag{250}$$

The summation may be carried out along *any path.*

The quantity $E = \Sigma f \cos e \Delta s$ is called the *electromotive force,* or the *difference of electric potential* between the points p and p'. E is evidently the work per unit charge done in carrying electricity from p to p'.

* This is true only when there is no changing magnetic field in the region.

Unit difference of potential or of electromotive force. — In conformity with equation (249), two points are said to be at unit difference of potential (or to have unit e. m. f. between them) when one erg of energy is required to carry a unit (electrostatic unit) charge from the one point to the other. The volt (work in *joules per coulomb of charge carried*) is of course a distinct unit of e. m. f. There are 300.2 volts in one electrostatic unit of e. m. f.

Electric potential at a point. — A region of zero potential being chosen arbitrarily, the potential V, at a given point, is defined as the *electromotive force*, or *difference of potential*, between that point and the region of zero potential. The earth is ordinarily chosen as the zero of potential. It is sometimes convenient to choose the region which is at an infinite distance from charges which are being studied as the zero of potential. Electric potential is a scalar quantity (a distributed scalar), and its gradient is intensity of field. (See Chapter I.)

468. Potential at a point distant r from a concentrated charge, or at a distance r from the center of a uniformly charged sphere. — Let it be required to determine the potential at a point p (Fig. 254), distant r from a concentrated charge Q. This

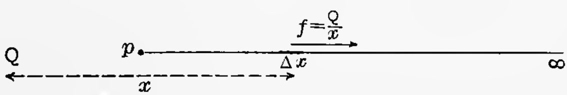

Fig. 254.

potential at p is the e. m. f. between that point and infinity (region of zero potential), reckoned along any path one may choose. Let the straight path $Q\infty$ be chosen. Then

$$E = \sum f \cdot \cos e \cdot \Delta s,$$

reckoned along this path, is the desired potential at p. Let Δx be the element of the path distant x from Q. Then, since

$f = \frac{Q}{x^2}$ (from equation (240)), and e is everywhere zero, the expression for E becomes

$$E = Q\sum_{r}^{\infty}\frac{1}{x^2}\Delta x = Q\int_r^{\infty}\frac{1}{x^2}dx = \frac{Q}{r}.$$

That is, the potential V, at a distance r from a concentrated charge Q, is

$$V = \frac{Q}{r}. \tag{251}$$

This equation also expresses the potential at an external point distant r from the center of a uniformly charged sphere; for, as has been shown in Art. 457, the charge on such a sphere acts as if it were concentrated at the center.

469. The potential, at a point due to a distributed charge, is, by the principle of superposition, equal to the sum of the potentials at the point due to the respective small parts of the charge. That is,

$$V = \sum\frac{\Delta Q}{r}, \tag{252}$$

in which V is the potential at a point p, and r is the distance of the small charge ΔQ from p.

470. Equipotential surfaces. — Imagine a surface to be drawn through a given point in an electric field, so as to be everywhere perpendicular to the lines of stress in the field. All points in such a surface are at the same potential; for, choosing a path between any two points ϵ is everywhere a right angle and the expression (250) is zero for this path. Such a surface is called, in consequence, a surface of *equipotential*. Any surface which is everywhere at right angles to an electric field is an equipotential surface. Thus the surface of a charged conductor is always an equipotential surface. A region in which there is no electric field is everywhere at the same potential. The interior of a charged conductor has everywhere the same potential as

its surface. The potential of a charged conductor is an important quantity in the specification of its electrical state.

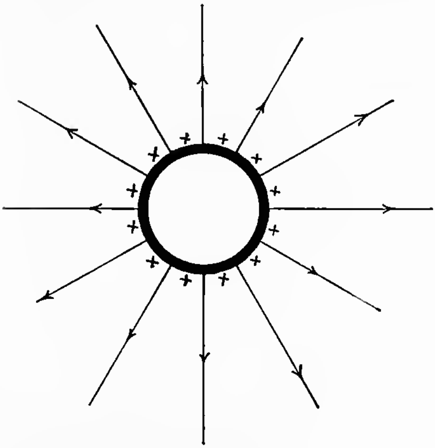

Fig. 255.

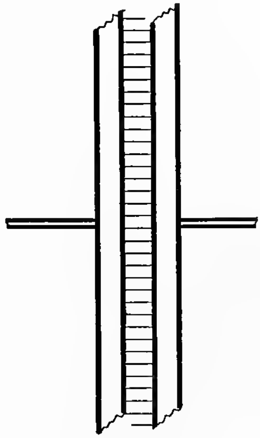

Fig. 256.

The following are some typical examples of lines of force and of equipotential surfaces.

(*a*) Field around a uniformly charged sphere (Fig. 255).

(*b*) Field between parallel charged cylinders (Fig. 256).

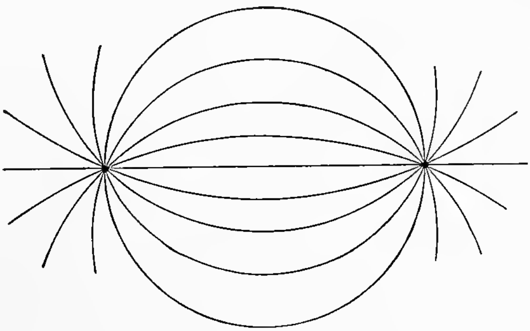

Fig. 257.

(*c*) Field due to two concentrated charges (Fig. 257.)

(*d*) Field due to a concentrated charge near an infinite conducting plane (Fig. 258).

471. Electric images. — *Proposition.* The surface of a charged conductor is an equipotential surface, and perpendicular to the lines of electric stress, which emanate from it (see Art. 464). It follows that any equipotential surface may be replaced by a thin metal shell or sheet of the same shape without disturbing the electric field on either side of the sheet. The sheet of metal will always be closed, or it will extend to infinity, so that the electrical fields on the two sides of it will be independent of each other (Art. 460). Therefore the electric field on one side of the sheet may be obliterated without affecting the field on the other side.

Example. — The equipotential surface midway between two equal and opposite charges, is a *plane.* If a plane sheet of metal be put in place of this equipotential surface, one of the charges

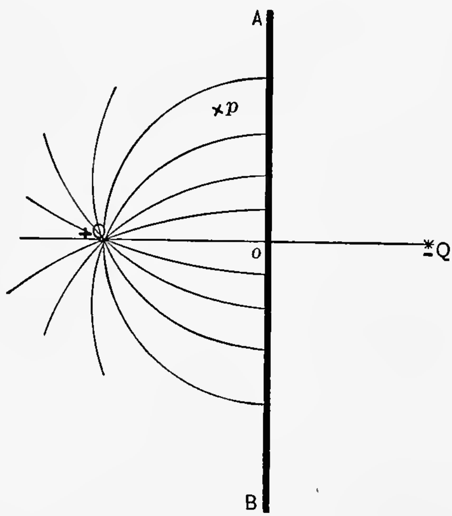

Fig. 258.

may be obliterated, leaving the field on the other side just as it was before. This is shown in Fig. 258. Now the electric field at a point p (Fig. 258) is of course the same as that which would be produced at p by the two charges $+Q$ and $-Q$. The

charge $-Q$, situated on the opposite side of the plane from $+Q$, and at the same distance from the plane as $+Q$, is called the *electrical image of* $+Q$ in the plane. This principle of replacing equipotential surfaces by conducting sheets furnishes easy solutions of the distribution of electric charge for a great many simple cases.*

* See J. J. Thomson's Elements of Electricity and Magnetism, pp. 138–183; also Maxwell, Treatise on Electricity and Magnetism, Vol. II. Chap. XI.

CHAPTER IX.

ELECTROSTATIC CAPACITY; ELECTROMETERS.

472. Mutual relation of two conductors. — Let A (Fig. 259) be an insulated conductor having a charge Q. Let all other conductors in the neighborhood be connected together and to earth, so that together they constitute a single conductor, of which the potential is zero. If A is charged positively the neighboring bodies will be charged negatively by influence. The potential V at any point p is (equation (252)) $V = \Sigma \frac{\Delta Q}{r}$. If the charges everywhere be increased n times, then each term of this sum, $\Sigma \frac{\Delta Q}{r}$, *will be increased n-fold, so that the potential everywhere will be increased n-fold.* The conductor A being previously at potential E, will be now at potential nE, while the remaining bodies being previously at potential zero will still have that potential. The charge everywhere being increased n-fold, the charge on A will be nQ. *Therefore the potential E of a conductor is proportional to its charge Q.* This relation is expressed by means of the equation

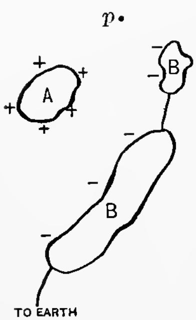

Fig. 259.

$$Q = JE, \tag{253}$$

in which Q is the charge of a conductor, E its potential. The proportionality factor J is called the *electrostatic capacity* of the body. The above statement and the equation $Q = JE$ would need to be somewhat modified were there more than *two bodies* which mutually influence each other.*

* For a discussion of the mutual influence of any number of bodies, see Maxwell, Treatise on Electricity and Magnetism, Vol. I. Chap. III.

473. The unit of electrostatic capacity. — In conformity with equation (253) a body is said to have unit electrostatic capacity when it carries one unit charge at unit potential. By equation (251) this charge is easily seen to be that of an isolated sphere of one centimeter radius. The dimensions of J are *length*, simply, so that J is expressed in *centimeters*. These dimensions are, however, fictitious and have no essential significance. (See Chapter V.)

A practical unit of capacity largely used in experimental work in electricity is the *farad*, which is the electrostatic capacity of a body which will hold one *coulomb* of charge at a potential of one volt. The farad is equal to 9.01×10^{11} "electrostatic" units of electrostatic capacity as here defined.

474. Energy of charge. — Consider an insulated body with charge Q and potential E, all other bodies being connected to earth, and at zero potential. The energy of such an electrical system is

$$W = \tfrac{1}{2} QV. \tag{254}$$

Since $V = \dfrac{Q}{J}$ (equation (253)), we may write equation (254) in the form

$$W = \frac{1}{2}\frac{Q^2}{J}. \tag{255}$$

Proof. — Let the charge Q be increased by a small amount ΔQ which is brought from the region of zero potential. To do this, an amount of work $\Delta W = E \cdot \Delta Q$ must be done (equation (249)). But $E = \dfrac{Q}{J}$ (equation (253)), so that $\Delta W = \dfrac{1}{J} \cdot Q \cdot \Delta Q$. Therefore, to increase the charge from zero to Q requires an amount of work $W = \dfrac{1}{J}\sum Q\Delta Q = \dfrac{1}{J}\displaystyle\int_0^Q Q \cdot dQ = \dfrac{1}{2}\dfrac{Q^2}{J}$. Q.E.D.

By a slight elaboration of this proof, it can be shown that, to obtain the energy of any system of charges, the charge at each point is multiplied by its potential, and half the sum of such products is taken. The expression for this operation is

$$W = \tfrac{1}{2}\sum V \cdot \Delta Q. \tag{256}$$

475. Condensers. — An arrangement, consisting of two conductors, one of which is connected with the earth, while the other is so placed with reference to the former as to give a large electrostatic capacity, is called a condenser.

In its simplest form (the air condenser), the condenser consists of two parallel disks (Fig. 260), mounted upon supports of glass or vulcanite. The supports have freedom of motion in the direction of the common axis of the disks, by which means the thickness of the air space between them is adjustable. The

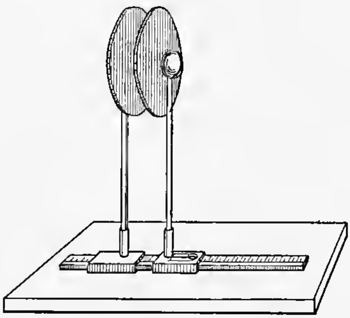

Fig. 260.

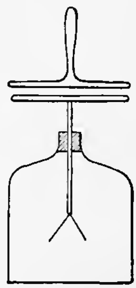

Fig. 261.

increase in the electrostatic capacity between parallel plates, as the distance between them diminishes, may be shown by means of the apparatus depicted in Fig. 261. This is, in reality, an air condenser, one of the plates of which is likewise the disk of a gold-leaf electroscope. If the electroscope be charged, and the upper disk, grounded by contact with the finger, be brought near, the divergence of the leaves will decrease. Upon the removal of the upper plate, they will return to their former angle. If, however, the upper plate be brought near and grounded, and the electroscope be charged to the same degree as before, the dilation of the leaves, when the upper plate is withdrawn, will be greatly increased. This increase will be more marked as the distance between the plates is reduced. *The capacity of the electroscope disk has been increased by the presence of the grounded plate in its neighborhood.*

Condensers are given various forms. To bring the plates very near to one another without danger of discharge, solid and liquid dielectrics are used instead of air. These have the further advantage of large specific inductive capacity (see Art. 481). To increase the area, plates are piled one upon another, as


Fig. 262.

shown in Fig. 262, and alternate plates are connected together. Whatever form be selected, however, the surfaces of the plates are almost always parallel. The most important forms of condenser are discussed in the following articles on ideal cases of electrostatic capacity (Arts. 476, 477, 478, and 479).

476. Electrostatic capacity of an isolated sphere. — The potential at an external point p, distance r from the center of a sphere with charge Q, is $V\cancel{E} = \frac{Q}{r}$ (Art. 457). This expression holds, however near p may be to the surface of the sphere, provided only that p is outside the sphere. Therefore *at* the surface of the sphere, $V\cancel{E} = \frac{Q}{R}$, where R is the radius of the sphere; therefore

$$Q = R\cancel{E}V. \tag{257}$$

Comparing this with equation (253), we see that *the electrostatic capacity of an isolated sphere is equal to its radius.*

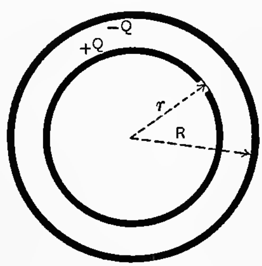

Fig. 263.

477. Electrostatic capacity of a sphere, of radius r, surrounded by a conducting spherical shell, of inside radius R, connected to earth. — Let Q be the charge on the sphere (Fig. 263); then $-Q$

is the charge on the inner surface of the shell. The potential, at any point, is of two parts, viz., the potential due to the charge $-Q$, and the potential due to the charge $+Q$. The potential *at all points* inside of the shell, due to $-Q$, is equal to the potential $\frac{-Q}{R}$ at the surface of the shell, due to that charge. The potential at the surface of the inner sphere, due to the charge Q, is $\frac{Q}{r}$. Therefore the total potential of the inner sphere is $E = \frac{Q}{r} - \frac{Q}{R}$, or $Q = \frac{1}{\frac{1}{r} - \frac{1}{R}} E$. Comparing this with equation (253), we have

$$J = \frac{1}{\frac{1}{r} - \frac{1}{R}} \tag{258}$$

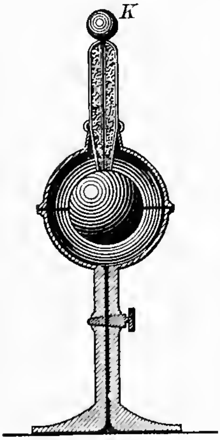

Fig. 264.

as the electrostatic capacity of the sphere under the stated conditions.

This proposition covers the case of the form of condenser used by Faraday * for the comparison of the specific inductive capacities of liquids. It consisted of a metal sphere (Fig. 264) surrounded by a hollow spherical shell, also of metal. The condenser was charged through the insulated rod which ended in the brass knob K. The region between the sphere and the shell was filled with the liquid, the inductive capacity of which was to be tested.

478. Electrostatic capacity per unit length of a cylinder of radius r coaxial with a hollow cylinder of inside radius R. — Let ν be the charge per unit length of the inner cylinder, then the electric field *between* the two cylinders at a distance x from

* Faraday, Experimental Researches, Art. 1189, etc., 1838.

the axis $f = \frac{2\nu}{x}$ (equation (244)). The difference of potential between the two cylinders may be calculated from the equation

$$E = \sum f \cdot \cos \epsilon \cdot \Delta s. \qquad (250\ bis)$$

Choosing a line perpendicular to the axis as the path, we have $\cos \epsilon =$ unity and $\Delta s = \Delta x$; so that

$$E = \sum \frac{2\nu}{x} \Delta x = 2\nu \int_r^R \frac{dx}{x} = 2\nu [\log_e R - \log_e r].$$

It follows that

$$\nu = \text{charge per unit length of cylinders} = \frac{1}{2(\log_e R - \log_e r)} E.$$

Comparing this with equation (253) we see that the *electrostatic capacity per unit length of the cylinder is equal to*

$$\frac{1}{2(\log_e R - \log_e r)}. \qquad (259)$$

This is an important case, because every insulated wire or cable which is protected by means of a metal sheath, or which is buried or submerged, is a cylindrical condenser of the type considered in this article. The capacity of such wires and cables plays an important part in their behavior when used for telegraphic or telephonic transmission, or in the transmission of power by means of alternating currents.

479. Electrostatic capacity per unit area of parallel metal plates with air between. — Let $+\sigma$ and $-\sigma$ be the surface densities of charge on the respective plates, *i.e.* charge per unit area of plates. Then, from Art. 462, $f = 4\pi\sigma$ is the intensity of electric field between the plates, and f is perpendicular to the plates and of the same intensity everywhere between them. (See Art. 458.) The difference of potential between the plates

is $V = fd$ from equation (250), d being the distance between the plates. That is,

$$V = 4\pi\sigma d \quad \text{or} \quad \sigma = \frac{1}{4\pi d}V. \tag{260}$$

Comparing this with equation (253) we see that *the electrostatic capacity per unit area of plates is* $\frac{1}{4\pi d}$.

This case applies approximately to the air condenser described in Art. 475; also, by taking into consideration the nature of the dielectric, it applies roughly to the Leyden jar.

480. The Leyden jar (Fig. 265) is a condenser constructed by coating the inside and outside of a thin glass jar with metal foil. The outside coating being connected to earth, the arrangement has a definite electrostatic capacity which is proportional to the area of the coatings, to the dielectric constant of the glass, and inversely proportional to the thickness of the glass. An ordinary Leyden jar, half-gallon size, has a capacity (J) of from 2000 to 4000 cm. (*i.e.* 2000 to 4000 electrostatic units (Art. 473)), according to the thickness and quality of the glass and the area of the coatings. In other words, such a jar has a capacity about equal to that of an isolated sphere 60 meters in diameter.

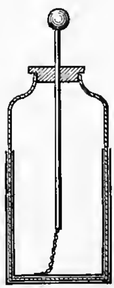

Fig. 265.

Such a jar has, however, a small capacity compared with the condensers built up of many layers, which were described in Art. 475. A condenser having a capacity of one microfarad ($= 0.000001$ farad), which is the usual size, is the equivalent of about 300 Leyden jars of the kind just described.

481. Specific inductive capacity or dielectric constant. — Faraday found the electrostatic capacity J of a condenser to vary with the dielectric between the plates. The ratio: *electrostatic capacity of a condenser, with a given dielectric between the plates, divided by its electrostatic capacity with air between the plates,* is called the *specific inductive capacity*, or the *dielectric constant* of the medium. Thus J, in the case of two concentric

spheres, or coaxial cylinders, or parallel plates, is about ten times as great as that given by the equations (258), (259), and (260) (for air), if the space between the spheres, the cylinders, or the plates is filled with heavy flint glass. The value of J for an isolated sphere is about ten times its radius if it is surrounded on all sides, to an indefinite distance, with heavy flint glass.

The specific inductive capacities of very many solids, liquids, and gases have been measured. Some of the most important values are given in the following table:

TABLE.

Specific inductive capacity of various substances.

Solids.		Liquids and Gases.	
Glass	$J = 3$ to 10	Water	73. to 90.
Sulphur	2.24 to 3.84	Olive oil	3.08 to 3.16
Vulcanite	2.50	Sperm oil	3.02 to 3.09
Paraffine	1.68 to 2.30	Turpentine	2.15 to 2.43
Rosin	1.77	Petroleum	2.04 to 2.42
Wax	1.86	Carbon disulphide . . .	1.81 to 1.60
Shellac	2.95 to 3.60	Hydrogen	0.9998 to 1.00026
Mica	4 to 8	Carbon dioxide	1.00080 to 1.00094
Quartz	4.5	Methane	1.0009

Electrometers.

482. The absolute electrometer. — A portion of area a (Fig. 266), of one plate of a parallel plate condenser, is hung from one end of a balance beam, by means of which the force may be measured with which this portion is pulled towards the other plate bb. In order that the portion a may be part of an indefinite plane sheet of metal, it is surrounded by a "guard plate" gg, which is in electrical connection with a. The force with which a is pulled by b may be computed as follows. It is a case of the attraction of charged parallel plates:

Let the surface density of charge on the two plates be $+\sigma$

and $-\sigma$. From Art. 463 we see directly that each plate is pulled towards the other with a force $2\pi\sigma^2$ *per unit area.* Substituting for σ the value $\frac{1}{4\pi d}E$ (Art. 479), we have

$$F = \frac{a}{8\pi d^2}E^2. \tag{261}$$

In this equation F is the attraction of area a of the two plates distant d from each other, when the difference of potential is E.

The observation of F, a, and d, in the case of the device shown in Fig. 266, makes possible the computation of E. The absolute electrometer is frequently constructed in the form shown in Fig. 267.

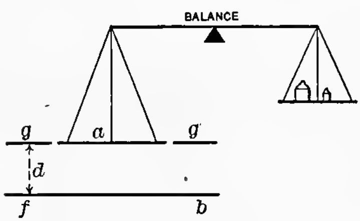

Fig. 266.

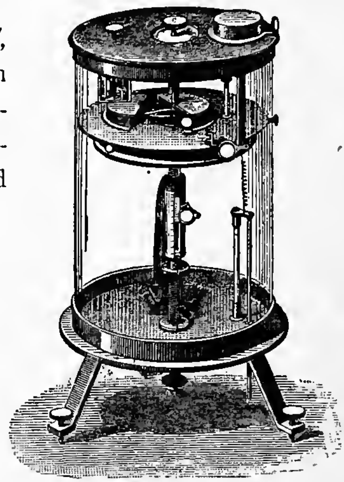
Fig. 267.

483. The quadrant electrometer. — The absolute electrometer is not very sensitive because the e. m. f. between the plates a and b must be considerable in order that the force F may be great enough to be measured. The *quadrant electrometer* is, however, extremely sensitive, and is constructed as follows:

A thin plate of metal pp, called the "needle" (Fig. 268), is suspended by a fiber,* which has sufficient torsional rigidity to give it a slight directive tendency. The needle hangs in the interior of a fixed flat cylindrical metal box, which is separated into four quadrants, $q_1q_1q_2q_2$. The quadrants q_1q_1 are connected

* Ordinarily a bifilar suspension is used.

by a wire c, and the quadrants q_2q_2 are connected by a wire d. The suspended plate has a stiff wire w projecting downwards

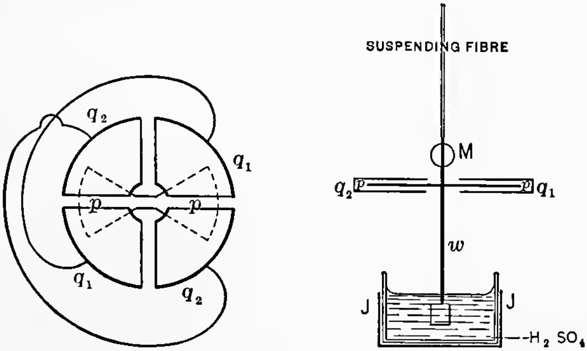

Fig. 268.

and carrying a small metal vane which dips into concentrated H_2SO_4. This acid forms the inner coating of a Leyden jar JJ, which being once charged is of sufficient capacity to keep the potential of the needle nearly constant in spite of leakage of charge. The metal vane in the acid serves to dampen the vibrations of the needle; and the acid further serves to keep the air dry inside the case, which surrounds the whole instrument. The arrangement of a simple form of quadrant electrometer is shown in Fig. 269. The quadrant electrometer is used in two ways, viz.:

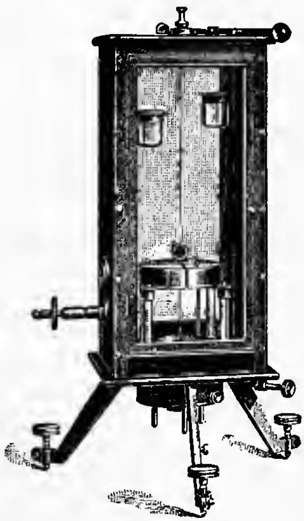

Fig. 269.

First arrangement (*for the measurement of very small potential differences*, or e. m. f.'s). — For this purpose the needle is charged, by means of an electrophorus, to a very high potential V_3. The e. m. f. to be measured is applied between the two pairs of quadrants q_1q_1 and q_2q_2, bringing them

to potentials V_1 and V_2, respectively. The torque T acting upon the needle in this case is proportional to $V_3(V_1 - V_2)$.* We may write

$$T = k_1 V_3(V_1 - V_2), \tag{262}$$

where k_1 is a constant. The needle turns until this torque $T = k_1 V_3(V_1 - V_2)$ is balanced by the torque $T = k_{11}\theta$ due to the twisting of the suspending fiber, θ being the angle of twist and k_{11} a constant. Therefore $k_1 V_3(V_1 - V_2) = k_{11}\theta$, or putting $\frac{k_{11}}{k_1} = k$, we have,

$$V_1 - V_2 = \frac{k}{V_3} \cdot \theta. \tag{263}$$

The quantity k/V_3 is called the *reduction factor* of the instrument. It can be determined by applying a known e. m. f., $V_1 - V_2$, for example the e. m. f. of a standard cell, and observing the deflection θ produced. When k/V_3 has thus been determined, the electrometer may be used to measure any other e. m. f.

Second arrangement (for the measurement of large e. m. f. and for measuring alternating e. m. f.'s). — For this purpose the needle and one pair of quadrants are connected together, and the e. m. f. $V_1 - V_2$ to be measured is applied between the needle and this pair of quadrants on the one hand, and the other pair of quadrants on the other hand, bringing them to potentials V_1 and V_2, respectively. The torque acting on the needle is in this case proportional to $(V_1 - V_2)^2$,* so that $T = k_1(V_1 - V_2)^2$. This torque produces a deflection θ of the needle, such that

$$(V_1 - V_2)^2 = k_{11}\theta,$$

or

$$V_1 - V_2 = k\sqrt{\theta}. \tag{264}$$

The quantity k is called the reduction factor of the instrument. It can be determined in a manner similar to that described above, and when once determined the instrument may be used to measure any other e. m. f.

* See J. J. Thomson, Elements of Electricity and Magnetism, p. 100.

CHAPTER X.

THE PHENOMENA OF DISCHARGE.

484. **Convective and conductive discharge.**—Consider the positive and negative charges at the ends of a Faraday tube. In order that these charges may disappear, it is necessary that the Faraday tube be annihilated. That is, it is necessary that the dielectric stress which constitutes the tube be either *let* down or *broken* down. This may occur by the charged surfaces moving towards each other until they are coincident; or by the actual breaking down of the mechanism which sustains the stress. In the former case we have what is known as *convective discharge*, in the latter we have *conductive discharge.* The term conductive discharge includes the processes of disruptive discharge, metallic conduction, and electrolytic conduction.*

485. **Convective charging and discharging.** (*a*) *Contact e.m.f.*—Any two bodies which differ from one another in chemical constitution, if left standing near one another, tend to settle down to a state in which they are at a *definite difference of potential.* If they are connected by a good conductor, as a wire, this state is reached very quickly. Thus two plates of zinc and copper, respectively, Fig. 270, are in electrical equilibrium only when at a potential difference e, of about one volt.

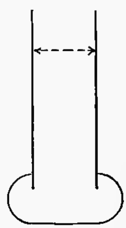

Fig. 270.

* It is probable that conductive discharge (disruptive, metallic conduction, and electrolytic conduction) takes place by molecular convection. However this may be, conductive discharge is an irreversible or sweeping process, and convective discharge, properly so called, is a reversible process. (See Vol. I. Chap. XII.)

If these plates are moved very close together, their electrostatic capacity $\left(\frac{1}{4\pi d} \text{ per unit area}\right)$ becomes very great, and if while in this position they are connected by a wire or allowed to stand for a long time, they will take on charge $\left(\frac{e}{4\pi d} = \sigma \text{ per unit area}\right)$. If the plates are now insulated and moved apart, the region between them will be left under intense electric stress, and the two plates will be found to be charged. As the plates are moved apart, their difference of potential increases greatly. This is evident from the equation $\sigma = \frac{e}{4\pi d}$. Since the plates are insulated, σ remains constant, and e must increase in direct proportion to d.

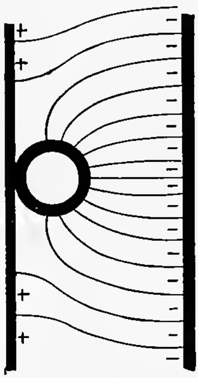

Fig. 271.

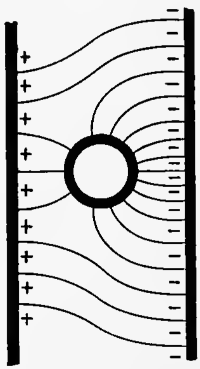

Fig. 272.

It is difficult to charge two metal plates highly by this process, because in withdrawing them from a position of very close juxtaposition, they are almost sure to come into contact with one another. The process of charging by contact and separation of non-conductors, such as rosin and fur, is probably similar to this for zinc and copper, except that no pains need be taken to insulate them during separation; and if one point of contact is separated before another, the liberated charge there cannot escape by being conducted along the charged surface. This charging by contact and separation, as has just

been pointed out, is a reversible process. The charged surfaces need only be brought together again to become discharged.

(*b*) *Convective discharge by means of a carrier.* — Figures 271, 272, 273, and 274 show the successive aspects of the electric field between two charged plates *AB*, as a metal ball which is at first in contact with *A* moves across and touches *B*. If the ball is made to move repeatedly to and fro, it continues to gather in the Faraday tubes and shorten them to zero as shown. This arrangement is in one (the essential) sense reversible, in that it converts electrical energy into mechanical energy, — not into heat.

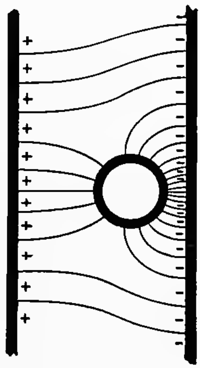

Fig. 273.

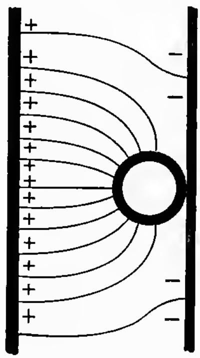

Fig. 274.

Insulated conductors surrounded by a mobile dielectric suffer discharge by means of the convective process just described. Portions of the fluid, and also whatever floating bits of dust it may contain, act as carriers. Along the edges of highly-charged conductors and near points, especially, convection currents of considerable strength arise from this cause. The air currents thus produced are used in many familiar experiments, such as that of the point and candle and of the electric whirligig.

(*c*) *Convective electric machines.* — The revolving doubler, the Töpler-Holtz machine, and the Wimshurst machine are arrange-

ments which, in their ordinary use, generate charges by convection. They are all reversible, and may be used as motors if run backwards and furnished with charge from some outside source. The frictional electric machine is also a convective electric machine, but not mechanically reversible because of the unavoidable friction of the pads on the glass, and also because these pads leave the glass positively charged, no matter on which side it leaves them.

486. Disruptive discharge. — Of the three kinds of conductive discharge, metallic conduction is treated under the head of resistance (Chapter IV.), and electrolytic conduction in the chapter on electrolysis (Chapter V.). In the present chapter, therefore, disruptive discharge, or dielectric conduction, will be considered.

It is shown in detail in Chapter VIII. how a rupture of the dielectric mechanism (for we can only picture it as a mechanism) along any line connecting two charged bodies enables the entire surrounding region to free itself from electric stress, just as a stretched piece of india-rubber may be relieved by cutting it across. Such a stretched piece of rubber might also be relieved by bringing the ends together. This process is distinct from that of relief by cutting. Likewise the conventional idea of electric charges moving along a rupture in a dielectric, or along a wire carrying current, is foreign to the conception of discharge by the breaking down of the dielectric.

487. The electric spark. — Let A and B (Fig. 275) be two conductors, surrounded by any dielectric, on which charge is being collected, for example, from an electric machine. The electric stress in the dielectric between A and B becomes more and more intense, as the charges increase, until an *electric spark* is formed between A and B. The duration of the spark is extremely short, and immediately after it A and B are found to be almost if not entirely discharged.

The extreme brevity of duration may be shown by observing image of the spark in a revolving mirror, or by photographing it upon a moving plate. For ordinary speeds the image appears as though the mirror, or the plate, had been at rest. By greatly increasing the speed, however, it is possible to show that the spark is not instantaneous, and to measure its duration.

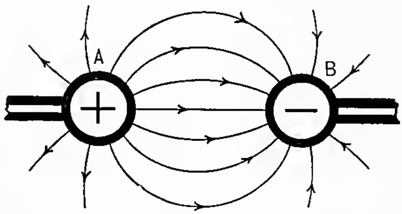

Fig. 275.

The method reveals the fact that the spark, in spite of its exceeding brevity, is often a complex phenomenon.

The electric spark shows much the same characteristics in all homogeneous dielectrics, — in glass, oil, rosin, air, or any gas (not too highly rarefied). In case of solid dielectrics, the path of the spark is found to be marked by a line, along which the material has been reduced to impalpable powder. In fluid dielectrics the indications are that molecular disintegration occurs along the path of the spark. The electric stress, much of which is no doubt sustained by the material of the dielectric, reaches a value, as the charges on A and B are increased, which the dielectric cannot sustain, and a breakdown of the structure (molecular) of the material is the result.

Consider a given electrical machine. This when in action generates charge at nearly a constant rate. As this charge flows into the knobs of the machine, it increases the potential between them until the air between is broken down, producing a spark and leaving the knobs nearly discharged. The frequency of the sparks is, therefore, determined by the amount of charge which must collect on the knobs before the potential

difference has reached the sparking point. This amount of charge is equal to the product of the sparking potential difference multiplied by the electrostatic capacity of the knobs. Therefore the frequency of sparking is increased by reducing the spark length (producing a decrease in the sparking potential difference) and by reducing the electrostatic capacity of the knobs, for example, by disconnecting the Leyden jars which are ordinarily connected with them.

488. Path of the spark. — As may be seen from Fig. 276, the path of the spark through air is not a straight line. Its vagaries are due chiefly to floating dust particles, the action of which is touched upon in Art. 489. The same tendency to

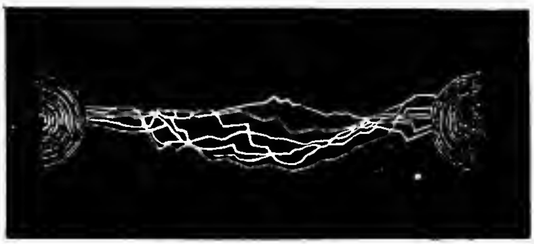

Fig. 276.

form a crooked path is seen in the case of lightning, and indeed wherever the electric discharge takes place through heterogeneous media.

Not only is the path of the spark crooked, but the successive sparks between any two conductors follow different paths. This tendency is also illustrated in Fig. 276, which gives the sparks occurring in one second between the terminals of a Holtz machine. The cut is from a photograph upon a stationary plate.

489. Electric strength of dielectrics. — For every dielectric there is a well-defined intensity of electric field, or electric stress, which is the greatest the dielectric can sustain without

rupture. Thus dielectrics are said to have definite *electric strengths*. The electric strength of a dielectric is specified by giving the intensity of electric field (ordinarily in volts per centimeter) which produces rupture. In such a case as is exhibited in Fig. 275, the electric field is nonhomogeneous, being of different intensities at different points, and the length of spark is consequently not related in a simple manner to the difference of potential between A and B.* Consider, however, two charged bodies A and B, Fig. 277, having flat portions facing each other. If the corners of A and B are rounded, as shown, the electric field will at no point be more

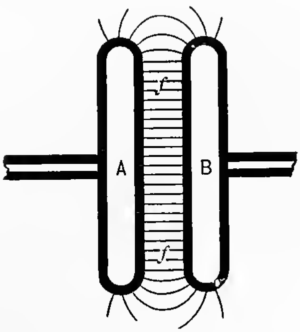

Fig. 277.

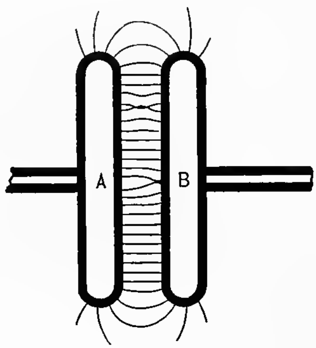

Fig. 278.

intense than it is in the region ff, between the plates where the field is homogeneous. In this region the field intensity is $f = \frac{E}{d}$, E being the difference in potential between A and B, and d the distance between the flat surfaces. If charge is collected upon AB until a spark is formed, and the corresponding value of E is measured, then $f = \frac{E}{d}$ gives the electric strength of the dielectric between A and B. The least roughness of the surfaces of A and B, or of clinging or floating particles of dust, produce great variations in the value of f for which a spark

* See J. J. Thomson, Elements of Electricity.

is formed. The action of these irregularities of surface and of floating particles is clearly shown, but somewhat exaggerated, by Fig. 278, in which a represents a floating particle and p a point projecting from the plate B. The intensity of the field near the point and near the ends of the particle a is much greater than the *average* intensity between A and B, which average intensity it is which is given by the ratio $\frac{E}{d}$. The dielectric will of course begin to give way at these places a and p, when the field intensity there has reached the breaking value.

The following table of the dielectric strength of various substances is from the measurements of Macfarlane and Pierce.*

TABLE.

Dielectric strength of various media.

Medium.	Strength in Volts per Centimeter.	Medium.	Strength in Volts per Centimeter.
Oil of turpentine . . .	94,000	Beeswaxed paper . .	540,000
Paraffine oil	87,000	Air (thickness 5 cm.)	23,800
Olive oil	82,000	CO_2 "	22,700
Paraffine (melted) . .	56,000	O "	22,200
Kerosene oil	50,000	H "	15,100
Paraffine (solid) . . .	130,000	Coal gas "	22,300
Paraffined paper . . .	360,000		

490. Maximum charge on a conductor. — As the charge on a conductor is increased, the electric field near its surface increases in intensity. This field may of course be more intense near some portions of the surface than near other portions. The conductor cannot hold a charge greater than *that for which the neighboring electric field, at its most intense point, begins to break down the surrounding dielectric.*

Case of a uniformly charged sphere. — Let σ be the surface density, then the total charge $Q = 4\pi r^2\sigma$, r being the radius of

* Physical Review, Vol. I. p. 165.

the sphere. The intensity of electric field at the surface of the sphere is $f = 4\pi\sigma$, from Art. 462. If f is the breaking intensity for the dielectric surrounding the sphere, $\sigma = \frac{f}{4\pi}$ is the greatest attainable surface density. This substituted in the equation $Q = 4\pi r^2\sigma$, gives

$$Q = r^2 f \tag{265}$$

in which Q is the greatest charge on a sphere of radius r, and f is the breaking intensity of the surrounding dielectric. In case of a gaseous dielectric this f is the electric strength of the gas near the surface * of the metal. Equation (265) holds for any case in which the surface density σ is the same over the whole sphere, *e.g.* when the sphere is surrounded by a concentric spherical shell. In case of an isolated sphere, $\frac{Q}{r}$ is the potential E of the sphere which, substituted in equation (265), gives, for the highest potential to which an isolated sphere of radius r can be brought, in a dielectric of strength f, the value

$$E = rf. \tag{266}$$

For example, for air under ordinary conditions as to temperature, pressure, and moisture, $f = 24{,}000$ volts per centimeter, or more, so that an isolated sphere of 10 cm. radius can be held at a potential of about 240,000 volts.

Case of uniformly charged circular cylinder. — Let σ $(= \frac{f}{4\pi}$, from equation (245)) be the surface density on the cylinder. Then $\nu = 2\pi r\sigma$ is the charge of the cylinder per unit length, so that

$$\nu = \tfrac{1}{2} rf \tag{267}$$

in which ν is the maximum charge per unit length of a uniformly charged circular cylinder of radius r in a dielectric of strength f.

* A gas seems to have excessive electric strength near the surface of a negatively charged metal conductor.

491. Progress of spark. — Let A and B (Fig. 279) be two bodies upon which charge has been collected until the electric intensity has reached the breaking point for the intervening dielectric. A *fracture* c will start in the region of greatest intensity. In air this region is always at the surface of the positively charged body, unless that body is much larger than the other. Air seems to have greater electric strength in the immediate vicinity of a negatively charged surface. Along this fracture the dielectric is a good conductor, either because it is intensely heated there, or because of the molecular disintegration which constitutes the fracture. The Faraday tubes on all sides of this fracture are therefore pushed into it, as shown by the arrows (see Art. 464), leaving a very intense field just at the end of the fracture, which is in this way continued across to the other body B. Thus a conducting line is established from A to B into which all the Faraday tubes on A and B are pushed by those still beyond and broken down. This movement of the Faraday tubes is attended with a momentum which, after all the original Faraday tubes have disappeared, generates reversed Faraday tubes along the conducting line. These reversed Faraday tubes spread out into the dielectric, constituting a reversed charge. When the momentum of the previous discharge is all used up in the production of this reversed charge, it in turn is discharged along the line of fracture, and the action of the previous discharge is repeated. The energy of the initial electric charge (see Art. 474) is soon lost as heat which appears along the line of the spark, so that after several oscillations the whole system comes to rest in a neutral condition. Those Faraday tubes which extend to a considerable distance from the charged balls A and B, in sweeping into the spark take on some such shape as that shown in Fig. 280. Eventually the branches at

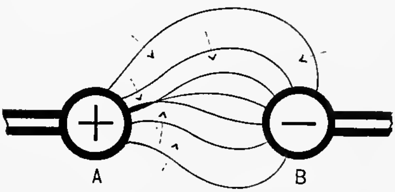

Fig. 279.

a coalesce, leaving an *endless* Faraday tube, which is no longer associated with any charged body. It passes off into space as a *wave*, with the velocity of light. (See Chapter XV.) In some cases quite a considerable portion of the energy of an electric charge is dissipated in this way. The oscillatory character of the disruptive discharge is easily shown by causing the image of an electric spark to fall upon a photographic plate which is traveling with great velocity, or by viewing it in a rotating mirror which is driven at very high speed, or by photographing the image from such a mirror.

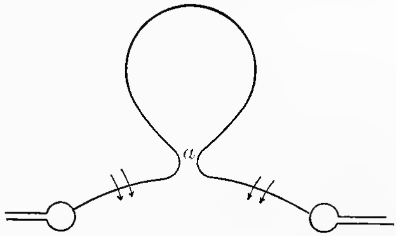

Fig. 280.

Sometimes the dielectric has time to heal between the oscillations, in which case the successive discharges follow independent paths, like those shown in Fig. 276. At other times it appears that the healing is incomplete and that the spark

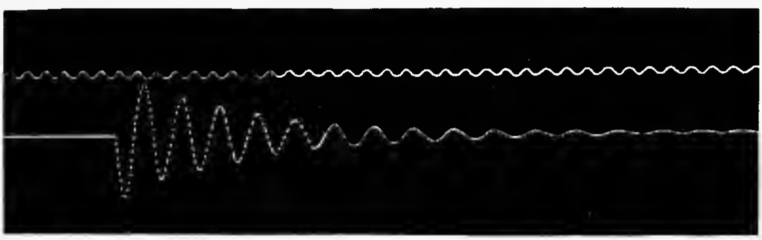

Fig. 281.

begins again at the positive pole and cleaves its way through the dielectric, which is still very weak, along the path of the previous discharge.

By means of a galvanometer of the D'Arsonval type, especially constructed with reference to securing an exceedingly high rate of vibration of the needle,* it is possible to follow

* See H. J. Hotchkiss, Physical Review, Vol. II.

the oscillations in the discharging current. Since the movements of the needle are far too rapid to be followed by the eye, a photographic trace is made. Figure 281 is the reproduction of such a record made by Mr. F. E. Millis. It represents the discharge of a condenser. The sinuous curve of small amplitude is a time-marking curve obtained by means of a mirror mounted upon a tuning fork.

492. The spark gauge. — The e. m. f. necessary to produce a spark between two polished metal balls or disks of a given size in air under given conditions, varies in a definite manner with the distance between them. If the e. m. f.'s required for different distances be once determined by observation, then any e. m. f. may be determined by measuring its sparking distance between the given balls or plates. The arrangement for making this measurement is called a spark gauge or *spark micrometer.* The spark micrometer is adapted only to high e. m. f.'s, and the results obtained by it are subject to large errors.

In Fig. 282 is shown a form of spark gauge employed by Steinmetz * in his investigation of the laws of sparking distance.

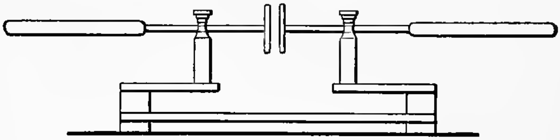

Fig. 282.

It consists of two insulated disks mounted horizontally and adjustable along the line of their common axis. They are polished, and their edges are carefully rounded.

493. The brush discharge. — The discharge in air from an isolated conductor, which is charged up to the limit set by the electric strength of the air (see Art. 490), is, in some respects,

* Steinmetz, Transactions of the American Institute of Electrical Engineers, Vol. X.

different in character from the spark discharge between two oppositely charged conductors which are not too far apart.

In this case, the lines of stress, before the rupture starts, diverge, as shown in Fig. 275, the intensity of the field growing less and less at greater and greater distances from the conductor. The rupture starting from the surface of the conductor very soon extends into the region where the field was originally much less intense than at the surface. Such Faraday tubes as have been pushed into the fracture and partially (*i.e.* a portion of their length) broken down, now radiate in a widely divergent bundle from the end of the fracture, as shown in Fig. 283. (Compare Fig. 283 with Fig. 279.) The result is,

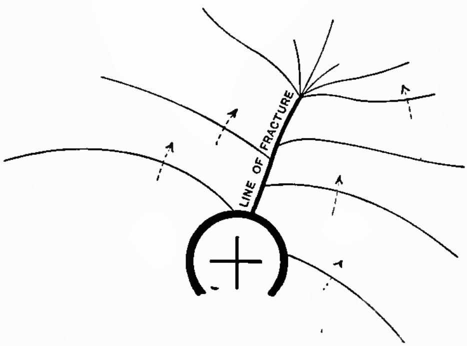

Fig. 283.

that the fracture divides into many branches, which penetrate into the surrounding air in the form of a *brush*. The brush discharge is formed most readily in a region where the lines of electric stress are widely divergent, as on pointed projections of a charged conductor. For some unknown reason, the *brush* forms more readily from a positively charged conductor than from one which is negatively electrified. The positive brush is very different in character from the brush on a negatively charged conductor. This is, no doubt, due to the properties of air, rather than to any difference between positive and negative charges.

494. Discharge by hot air or gas. — A hot gas is electrically very weak, and if the Faraday tubes from a charged conductor pass through such a gas, they break down. If the mass of hot gas is at a distance from the conductor, the Faraday tubes near the conductor will be left intact, and so also the charge on the conductor, the mass of gas becoming charged by influence, just as would a conducting body. (See Art. 465, Fig. 240.) If, however, the hot gas comes in contact with the charged surface, the Faraday tubes break down at the surface and leave that discharged.

495. The influence of pressure upon the disruptive discharge in gases. — The discussion given in the foregoing articles applies, in so far as it deals with gaseous dielectrics, to gases at ordinary pressures. When the pressure is reduced to a few centimeters of mercury, the appearance of the spark begins to change; the path becomes broad and ill-defined. The energy, which at greater pressures would have been expended along a nearly linear region, producing high temperatures and intense incandescence of the gas, is distributed throughout considerable volumes. The result is a weaker illumination with a color characteristic of each gas. This is known as the *Geissler discharge*. The effect changes continually, with diminishing pressure, in a manner the theory of which is but imperfectly developed. The electric strength of the gas diminishes rapidly to a minimum; then rises gradually, and approaches infinity as the pressure approaches zero. The precise law of these changes has not yet been determined.

Before exhaustion the discharge through the tube is in the form of a spark as in the open air. As the pressure in the tube is reduced, and the spark widens and becomes more and more nebulous, there is formed gradually a dark region, sharply defined, which surrounds the metal of the cathode. Beyond this *dark space* is a region which (for air) gives off a beautiful bluish violet light, called the *negative glow*. The

thickness of the dark space (measured normally to surface of the cathode plate) and of the negative glow both increase as the pressure decreases. Beyond the negative glow and extending to the anode is a luminous region called the *positive column*. This consists of a succession of light and dark regions or *striæ*. The distance between adjacent striations increases as the pressure diminishes. These striæ have in most cases an irregular motion along the tube, which often makes it difficult to distinguish them.

The above effects are exhibited by a vacuum tube at a moderate degree of exhaustion (pressure a few millimeters of mercury). Such a vacuum tube is called a *Geissler tube*.

When the exhaustion of a vacuum tube is carried to an extreme, leaving a pressure of a few thousandths of a millimeter of mercury, the dark space around the cathode expands until it fills the entire tube. The walls of the tube then show a brilliant green or blue luminescence, according as the tube is made of German glass or lead glass. A slight negative glow may remain in the portions of the tube remote from the cathode or near the anode.

The phenomena exhibited by such a highly exhausted tube were first studied by Crookes in England, and by Hittorf in Germany. Such a tube is called a *Crookes tube*.

496. Cathode rays. — If an object of any kind is placed in a Crookes tube, it is found to cast a sharp shadow (*i.e.* a spot where the wall is no longer luminescent) upon the wall of the tube; just as if the cathode were the source of rays which proceed in straight lines until they strike the walls of the tube, where they produce luminescence. Figure 284 shows a common form of Crookes tube for exhibiting this shadow effect. These *cathode rays* stream out from the cathode in a direction at right angles to its surface at each point. Thus a cathode in the form of a flat plate gives from its face a bundle of parallel cathode rays. A convex cathode gives a bundle of divergent rays, and

a concave or cup-shaped cathode gives a bundle of convergent rays, which concentrate at the center of curvature of the cathode plate, and then, if no obstacle prevents, diverge.

An object upon which the cathode rays impinge is heated, it may be to a very high temperature. Many substances emit light (without being perceptibly hot) when subjected to the action of the cathode rays. Such substances are said to be *luminescent*. For example, lead sulphate emits intense violet light; zinc sulphate emits white light; $MgSO_4 + 1\ \% \ MnSO_4$ emits intense red light under the action of the cathode rays. The glass walls of the Crookes tube are, as stated above, luminescent. Barium platinocyanide, magnesium platinocyanide, cal-

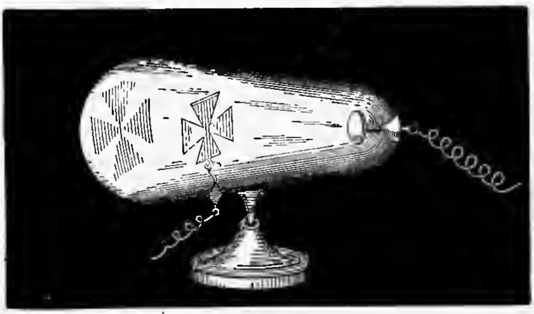

Fig. 284.

cium tungstate, and the various other salts which are used for the luminescent screens in studying the Röntgen radiations (see below), show brilliant luminescence under the direct action of the cathode rays.

The cathode rays pass quite readily through thin metal plates, especially through aluminum and other light metals, which are interposed in their path in the Crookes tube. Lenard, by using a Crookes tube of which a portion of the wall was made of thin sheet aluminum, got the cathode rays to pass through into the outside air. He found the rays capable of traversing twenty centimeters or more of atmospheric

air, of exciting luminescence, and of affecting the photographic sensitive plate.

The cathode rays exert a pressure upon an object upon which they impinge. Crookes mounted a small paddle wheel in a tube so arranged that the cathode stream would fall upon the paddles on one side, and cause the wheel to rotate.

The cathode stream is deflected to one side when it passes through a magnetic field; the direction of the stream, the direction of the field, and the direction of the deflection being mutually perpendicular. This is easily shown by placing a horseshoe magnet with its poles on opposite sides of the tube shown in Fig. 284. The shadow of the cross will be thrown up or down according to the arrangement of the magnet.

497. Crookes' theory of the cathode rays. — Crookes in his study of the electric discharge in high vacua was led to think of the discharge as taking place by convection. According to his view, molecules of the gas, perhaps dissociated, come into contact with the cathode, are charged negatively, and hurled off at a high velocity. These projected atoms do not often collide with each other because there are so few of them in the tube, but continue their rectilinear motion until they strike some obstacle. Such negatively charged moving atoms would be equivalent to an electric current towards the cathode, and in this way we may explain the deflection of the cathode rays by a magnetic field. (Compare Art. 353.) The heating action and force action of the cathode rays, and their action in exciting luminescence, seem to be pretty well explained by Crookes' conception; but this conception seems to be inconsistent with the fact that the cathode rays pass readily through thin metal plates.

498. Röntgen rays. — Objects upon which the cathode rays impinge not only become heated and luminescent (giving off ordinary light though not necessarily red hot), but they have

been found by Röntgen to emit a type of radiation which seems to be distinctly different from the cathode rays themselves, and different from ordinary light and from radiant heat. This type of radiation, the *Röntgen Rays*, as they are called, are best generated from a piece of platinum placed at the center of curvature of a cup-shaped cathode of a focusing tube (Fig. 285), but the rays are generated wherever the cathode rays impinge upon the walls of a Crookes tube, or upon any object contained therein. Any tube having a sufficient vacuum is therefore adequate for most purposes of demonstration. The Röntgen rays pass through all substances, but dense substances are less permeable than light ones. Thus dense bodies cast shadows. These shadows may be rendered visible by using a luminescent screen, in which case the screen exhibits a faint glow except in the deeper portions of the shadow. The Röntgen rays affect the photographic plate, and it is possible, therefore, to take a shadow photograph by means of them, and thus to render the effects permanent. Figure 286 is from such a shadow photograph or skotograph (so called) of the hand. It was taken by allowing a Röntgen ray shadow of the hand to fall upon a sensitized plate. The plate was then developed as in the case of an ordinary photograph.

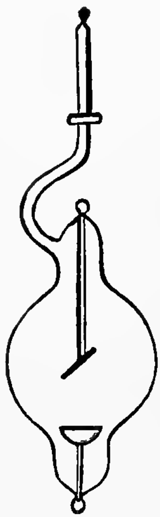
Fig. 285.

In a paper* published in 1891 von Helmholtz developed a new theory of the dispersion of light and discussed in detail the properties of ether waves (light) of extremely short wave-length. Professor J. J. Thomson has recently called attention to this remarkable paper, pointing out that these waves as conceived by von Helmholtz have *all the characteristics of the Röntgen Rays.* The projected particles which constitute the cathode rays in a Crookes tube seem to move at velocities com-

* Wiedemann's Annalen, Vol. 48.

parable to that of light,* and in striking the walls of the tube these atoms seem to be set in vibration, giving off ether waves of which the wave length is comparable to the dimensions of the atoms themselves.

Fig. 286. From a photograph taken in Berlin.

499. The electric arc. — When the discharge through the air at ordinary pressure or through a vacuum tube becomes intense enough to heat the gas to a very high temperature, the discharge assumes a flame-like character between the electrodes, which themselves become very hot, and a very considerable current may be made to pass with but a small e. m. f. Such a discharge is called the *electric arc* or the *arc discharge*. Figure 287, taken from a photograph, represents the arc discharge between two carbon rods in open air. In this case a current of about 10 amperes was flowing, and the e. m. f. between the carbons was about 50 volts. The arc discharge through a gas

* See Oliver Lodge, London Electrician, July 17, 1896.

at low pressure is easily shown by taking a tall U-tube of glass, filling it with mercury, and inverting with each leg in a separate cistern. A dynamo being connected, through a rheostat, to these cisterns, the circuit is momentarily completed by inclining the tube until it is filled with mercury, when, upon being brought again into an erect position, the space above the mercury becomes brilliantly luminous, being traversed by a current of four amperes or more, with an e. m. f. of 20 or more volts between the cisterns.

Fig. 287.

500. Chemical action of the electric discharge in gases. — The stress which is sustained by a gas (or any insulator) in an electric field seems to be such as to tend to break up the molecules of the gas; and the rupture which occurs at the time of discharge seems to be of the nature of a molecular disintegration along the path of the discharge.

The electric discharge through oxygen (or air) produces ozone, which can be recognized by its odor near any electric machine or induction coil in operation. It seems that the ordinary biatomic oxygen molecules ($O = O$) are broken up by the discharge, forming monatomic oxygen, which immediately recombines, forming mostly $O = O$, but also a certain amount of the less stable triatomic oxygen $\left\{\begin{array}{c} O \\ / \ \backslash \\ O\text{—}O \end{array}\right\}$ or ozone. All gases seem to be dissociated by the electric spark.

501. Electric vibrations; electric waves. — The disruptive discharge is an almost instantaneous relief for the electric stress in the immediate neighborhood. This relief passes out from the line of the discharge as a *wave*. In case the discharge is

oscillatory, as it often is, a *train of waves* is sent out, carrying off energy. The first experimental study of electro-magnetic waves was made by Hertz. He used at different times a variety of apparatus, of which the following is perhaps the form best adapted to the study of waves.

The vibrator consists of two brass rods *A* and *B* (Fig. 288), with a spark gap at *g*. These rods are connected with the secondary terminals of an induction coil as indicated. An impulse from the induction coil charges the rods *A* and *B* (oppositely) more and more until the air gap *g* breaks down, when the discharge surges back and forth until the energy of the charge is dissipated. This action is repeated with each impulse from the induction coil.

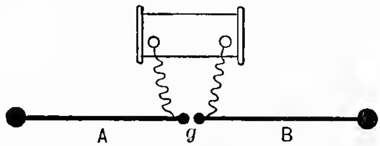

Fig. 288.

The resonator. — The electric waves are detected by means of an arrangement exactly similar to the vibrator, but with a shorter spark gap, and without connections to an induction coil. This arrangement, called the *resonator*, has the same period of oscillation as the vibrator, so that the action upon it of the train of waves from the vibrator is cumulative, causing it to vibrate in sympathy with the vibrator; just as one tuning fork vibrates in sympathy with a similar one which is set vibrating by a hammer blow. The vibrations of the resonator are indicated by minute sparks in its spark gap.

The reflectors. — The waves emanating from the Herz vibrator are very weak at any considerable distance, and their action upon the resonator is scarcely perceptible. Their action may be greatly intensified by the use of parabolic reflectors. The vibrator and the resonator are placed along the respective focal lines of two parabolic cylinders made of sheet metal. These are shown in vertical and horizontal sections in Fig. 289.

The resonator *CD* (Fig. 289) is arranged so that the spark

gap is behind the mirror, as shown at *g*. It is thus rendered the more easily visible.

Reflection of electric waves. — When the vibrator and resonator are arranged as shown in Fig. 289, a very distinct

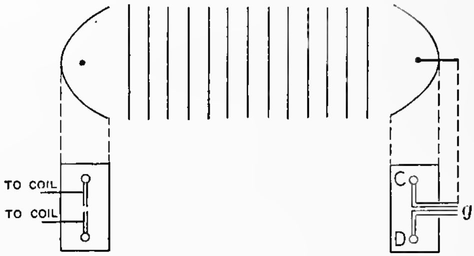

Fig. 289.

action on the resonator is produced when the vibrator is active, the waves emanating from the vibrator being concentrated upon the resonator by the action of the two parabolic reflectors.

When arranged as shown in Fig. 290, *AB* being a plane sheet of metal, and the angles ϕ being equal, a very distinct action on the vibrator is likewise produced.

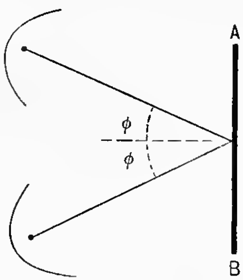

Fig. 290.

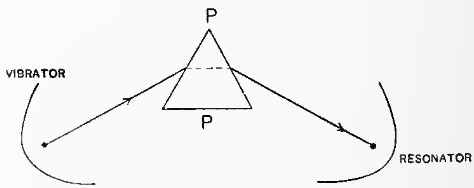

Fig. 291.

Refraction of electric waves. — When the vibrator and resonator are arranged as shown in Fig. 291, in which *PP* represents a large prism of asphaltum or paraffin, the resonator shows likewise very distinct action.

Polarization of electric waves. — A frame strung with a grating of fine metal wires acts as a good reflector for these

electrical waves, when the wires of the grating are parallel to the axis of the vibrator. In this case the grating allows almost no portion of the waves to pass through it. When the wires of the grating are at right angles to the axis of the vibrator, the waves pass through it without perceptible diminution in intensity and without perceptible reflection.

Stationary electric waves. — If the vibrator be faced towards a plane metal sheet AB (Fig. 292), the resonator, removed from its parabolic reflector, will be found to show no action near the wall at p. As it is moved away from the wall it will become more and more active.

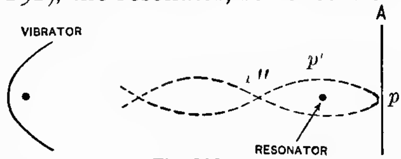

Fig. 292.

Passing a place of maximum activity at p', it will then come into a region of no activity at p'', and so on, as represented graphically by the dotted lines. The distance pp'' is the distance traveled by the electrical waves during one oscillation of the vibrator. If the period τ of the vibrator be determined experimentally, the distance $pp'' \div \tau$ gives the velocity of progression of the electric waves. This is found to be exactly the velocity of light, viz., $299 \times 10^8 \frac{\text{cm}}{\text{sec}}$.

This value accords also with the velocity of electric waves as calculated from electric data. The period of one oscillation of such a vibrator as described above is in the neighborhood of two or three hundred millionths of a second.

The Leyden jar as a vibrator and as a resonator. — Two Leyden jars may be made to act as vibrator and resonator respectively by arranging each as shown in Fig. 293, and adjusting the junction a of one jar until the two jars have the same period.

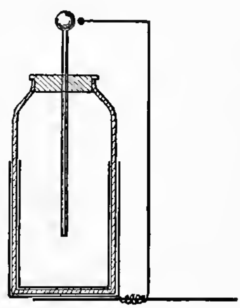
Fig. 293.

If one jar is then charged until the air

gap g breaks down, the discharge will be oscillatory, and waves will be sent out which will cause the other jar to oscillate in sympathy. If the jars are not placed too far apart, this sympathetic oscillation of the second jar will show itself by means of a feeble spark.

502. Tesla's induction coil.* — The following type of induction coil for the production of oscillatory sparks of high frequency is due to Nikola Tesla.

A helix PP of say ten to fifteen turns of wire is connected to the terminals CD of a large condenser AB with a spark gap at g. The condenser, connected to the secondary of a transformer (10,000 to 20,000 volts), is charged until the air gap at g

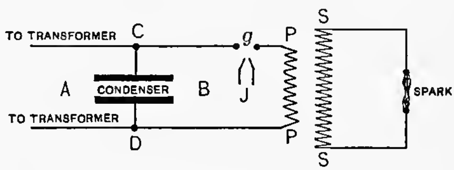

Fig. 294.

breaks down, when the charge of the condenser surges back and forth through the helix PP until the energy of the charge is dissipated. A jet of air issues from a nozzle J, blowing away the air which has been heated and dissociated by the spark. Then the charge upon the condenser again increases until a fresh discharge occurs. The successive discharges may be as frequent as several hundred or several thousand per second, and the oscillations of each discharge may be at the rate of several hundred thousand per second.

Another helix, SS, of several hundred turns of wire, is arranged with its axis coinciding with the axis of the helix PP (not so shown in the figure). The coils PP and SS constitute the primary and secondary coils of an induction coil or trans-

* See Art. 556.

former. The rapidly oscillating current in *PP*, due to the discharge of the condenser, induces enormous e. m. f.'s in *SS*, producing a long spark between the terminals of *SS*.

The coils *PP* and *SS* have to be very highly insulated, for which purpose it is usual to place all the coils in an oil bath. A very striking property of the discharge from *SS*, as of any oscillatory discharge of very high frequency, is that it traverses only the layers near the very surface of a wire, or any conductor through which it passes, and it may be in consequence passed through (over) the human body with impunity.

The condenser *AB* should be arranged so that one may change its electrostatic capacity at will. Such a change alters the period of oscillation of the discharge through *PP*, and the operator may thus bring the oscillations of *AB* into unison with the proper oscillations of *SS*, under which condition the action of the arrangement is most intense.

CHAPTER XI.

MAGNETISM IN IRON.

503. Propositions preliminary to the discussion. — When a bar of iron is placed in a magnetic field it becomes a magnet. For the purpose of studying this action of a magnetic field upon iron, it is necessary to be able to subject iron to the action of magnetic fields of various known intensities. Such magnetic fields are ordinarily produced by a coil of wire carrying electric current, and it is necessary to discuss, at this point, the action of coils in the production of magnetic field. The following articles (504 to 509) are thus preliminary to the discussion of magnetism in iron.

504. Proposition. — *The work, ΔW, done on a circuit in maintaining a current i in that circuit, constant while the magnetic flux* through the opening of the circuit changes by the amount ΔN is*

$$\Delta W = i \cdot \Delta N. \tag{268}$$

Proof. — The rate of change of flux, $\frac{dN}{dt}$,† is a counter-electromotive force in the circuit, and this e. m. f. multiplied by the current gives the rate, $\frac{dW}{dt}$, at which work must be done in forcing the current against this e. m. f. That is, $\frac{dW}{dt} = i\frac{dN}{dt}$. Therefore, multiplying both members by Δt, we have

$$\Delta W = i \cdot \Delta N.$$

* Magnetic flux is analogous to electric flux. See Art. 452. Compare also Arts. 320–325, Chapter I. The first part of Chapter XII. should be studied as an introduction to the present chapter

† See Art. 538, Chapter XII.

505. Proposition 1. — *The magnetic flux N through a closed loop from a neighboring magnetic pole of strength m is*

$$N = \omega m, \tag{269}$$

in which ω is the solid angle subtended by the loop as seen from the pole.

Definition of solid angle of a cone. — Describe a sphere of radius r, with its center at the apex of the cone. Let A be the area of the portion of this spherical surface which is inside the cone. Then $\frac{A}{r^2}$ is a constant for the given cone, and it is called the solid ω angle of the cone. That is,

$$\omega = \frac{A}{r^2}. \tag{270}$$

The solid angle of a cone may have any value from zero, when the cone is very acute, to 4π, when the entire area $4\pi r^2$ of the sphere is inside the cone. This method of reckoning solid angles is analogous to the reckoning of plane angles as *arc divided by radius*. A plane angle reckoned in this way may have any value from zero to 2π.

Remark. — The solid angle subtended by a closed loop as seen from a given point, is the solid angle of a cone formed by drawing straight lines from the point to all parts of the loop.

Proof. — Describe a sphere of radius r about the magnetic pole as a center. Let A be the area of the portion of this spherical surface, which is inside a cone formed by drawing straight lines from the pole to loop. The intensity of the magnetic field at this spherical surface, due to the pole, is $f = \frac{m}{r^2}$ (from equation (193)). This, multiplied by the area A, gives the magnetic flux through that area. Now the lines of force radiate straight from the pole, so that the magnetic flux through the area A is the same as the flux through the loop. That is, $N = \frac{m}{r^2} \cdot A$. But $\frac{A}{r^2} = \omega$; therefore $N = \omega m$.

Remark. — Any movement of the pole which changes ω by the amount $\Delta\omega$, produces a change in N, such that $\Delta N = m \cdot \Delta\omega$ (from equation (269)). If the pole is carried once around any path which links once with the loop, coming back to its initial position, then the change in ω is 4π, and this movement of the

pole will therefore produce a change of flux, $\Delta N = 4\pi m$, through the loop.

506. Definition of magnetomotive force, or difference of magnetic potential. — Let W be the work done by the magnetic forces upon a pole of strength m while it is carried along a given path from one point to another in a magnetic field. The ratio $\frac{W}{m}$ is called the magnetomotive force (m. m. f.) along the path or the difference of magnetic potential between the points. That is,

$$\text{m. m. f.} = \frac{W}{m}. \qquad (271)$$

507. Proposition. — *The m. m. f. along a path in a magnetic field is the line integral* (see Art. 315, Chap. I.) *of the magnetic field along the path.*

Proof. — Let pp' (Fig. 295) be two points in a magnetic field. Let Δs be an element of a path between pp', and let f be the intensity of the magnetic field at Δs. A magnetic pole of strength m, if placed at Δs, will be acted upon by a force $F = mf$ (192), and the resolved part of this force in the direction of the element Δs is $mf \cos \epsilon$, which, multiplied by Δs, gives the work ΔW done by the force F while the pole is moved along Δs. That is,

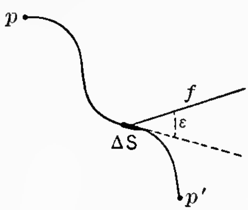

Fig. 295.

$$\Delta W = mf \cdot \cos \epsilon \cdot \Delta s.$$

The total work done while the pole is moved from p to p' is

$$W = \sum mf \cdot \cos \epsilon \cdot \Delta s \quad \text{or} \quad W = m \sum f \cdot \cos \epsilon \cdot \Delta s.$$

Therefore the m. m. f., $\frac{W}{m}$, along the path pp' is

$$\text{m. m. f.} = \sum f \cdot \cos \epsilon \cdot \Delta s. \qquad (272)$$

That is, the magnetomotive force along any given path in a magnetic field is the line integral of the field along the path.

Corollary. — The m. m. f. along a path of length l, parallel to a field of uniform intensity f, is

$$\text{m. m. f.} = fl. \tag{273}$$

In this case ϵ is everywhere zero, and $\cos\epsilon =$ unity; further, f being everywhere of the same value,

$$\text{m. m. f.} = \sum f \cdot \Delta s = f \sum \Delta s = fl.$$

508. Proposition. — *The magnetomotive force along any path which links once with a circuit carrying an electric current i is*

$$\text{m. m. f.} = 4\pi i. \tag{274}$$

Proof. — Let a magnetic pole of strength m be carried once around the path which links once with the circuit. This produces a change of magnetic flux through the opening of the circuit of $N = 4\pi m$ (by Art. 505), so that an amount of work $W = 4\pi mi$ must (by equation (268)) be spent on the circuit on account of this change of flux. When the pole reaches its initial position, *everything is as at first, and the work $W = 4\pi mi$ which has been done on the circuit must have escaped from the system. It is the work which has helped push the pole along its path.* Therefore $W = 4\pi mi$ is the work which the magnetic field due to the current i has expended in pulling the pole along the path, and the m. m. f., or $\frac{W}{m}$, along the path is $4\pi i$.

Corollary. — The m. m. f. along a path which links Z times with a circuit (or with which the circuit links Z times) carrying an electric current i, is

$$\text{m. m. f.} = 4\pi Z i. \tag{275}$$

Consider the magnetic circuit of a dynamo, indicated by the dotted line in Fig. 296. Coils of wire are wound on *aa aa*, say

Z turns of wire in all. If a current i is sent through these coils, the m. m. f. around the dotted line, with which the wire links Z times, will be $4\pi Zi$. If the current is expressed in amperes, its numerical value will be ten times greater, so that the m. m. f. will be $\frac{4\pi}{10} Zi_{\text{amp.}}$

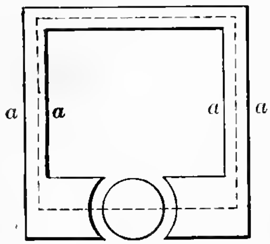

Fig. 296.

The product $Zi_{\text{amp.}}$ expresses the m. m. f. in terms of a unit called the *ampere-turn, which is the m. m. f. along a path which links once with a circuit carrying one ampere of current.* A magnetomotive force expressed in ampere-turns must be multiplied by $\frac{4\pi}{10}$ to reduce it to c. g. s. units.

509. Proposition. — *The magnetic field inside a long uniformly wound coil of wire is uniform, and its intensity f is*

$$f = 4\pi ni, \tag{276}$$

in which n is the number of turns of wire in each centimeter of length of the coil, and i is the current flowing.

Proof. — The intensity of field in the axis of such a coil can be derived without much difficulty from equation (201), but we shall give a proof based upon the equation (275), viz.,

$$\text{m. m. f.} = 4\pi Zi.$$

Consider an indefinitely long coil of wire AB (Fig. 297). The magnetic field inside this coil, and also the magnetic field outside of it, must be everywhere *parallel* to the coil by symmetry. Consider a path $abdc$, such that the length $ab = l$. This path links with nl turns of wire on the coil so that the m. m. f. around this path must by equation (275) be

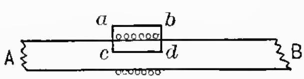

Fig. 297.

$$\text{m. m. f.} = 4\pi nli. \tag{a}$$

The portions ac and bd of this path being everywhere perpendicular to the magnetic field do not contribute to the m. m. f. Further, the m. m. f. along cd is fl, where f is the intensity of magnetic field along cd, and this contribution fl to the m. m. f. along the portion cd must be the same, however far it may be inside the coil, so that f must have the same value everywhere inside the coil. That is, the field inside the coil is uniform. Similarly, the field outside the coil must be uniform.

Let f' be the field intensity outside of the coil, and f the field intensity inside of it, then the m. m. f. along the path $abcd$ is $l(f-f')$, so that by equation (a) we have:

$$l(f-f') = 4\,\pi n l i. \tag{b}$$

The magnetic flux through the opening of the coil is $\pi r^2 f$, where r is the radius of the coil, and the magnetic flux back outside of the coil is $f'A$, where A is the indefinitely large area of a plane perpendicular to the coil. These two fluxes must be equal, for the one is merely the return flow of the other. Therefore since A is indefinitely great, f' must be indefinitely small or zero. So that equation (b) reduces to $f = 4\,\pi n i$.

Remark. — When a sample of iron is to be subjected to a magnetic field of known intensity, it is usually placed in a long coil having a known number of turns n per unit length, and through which a known current i is passed, so that $f = 4\,\pi n i$ is known. Such a long coil of wire is often called a *solenoid*.

Magnetism in Iron.

510. Magnetization. — An iron rod NS (Fig. 298) placed in a magnetic field f, as shown, becomes a magnet. For example, a bar of soft iron or of mild steel when held in the direction of the earth's magnetic field shows north polarity at one end and south polarity at the other. If the bar is turned end for end, its magnetism is reversed. A sharp blow with a hammer renders the bar

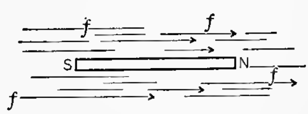

Fig. 298.

much more susceptible to the influence of the weak magnetic field of the earth. The polarity of the bar is easily indicated by a small magnetic needle hung by a silk fiber or supported by a jewel and point. This action of a magnetic field upon iron is called *magnetization.*

511. The electromagnet. — The action of magnetization is most strikingly shown by the comparatively intense magnetic field inside of a coil of wire carrying electric current. Thus an iron rod *NS* (Fig. 299) wound with an insulated wire becomes a very strong magnet when current is sent through the wire. An iron rod wound with wire in this way is called an *electromagnet. That end of the rod becomes a north pole, towards which a right-handed screw would move if turned in the direction in which the current circulates around the rod.*

Fig. 299.

A common type of electromagnet is shown in Figs. 300 and 301. It consists (Fig. 300) of a bent iron rod with winding distributed along it or at the ends; or (Fig. 301) of two straight rods *cc*, with a connecting yoke *YY* of iron. In these forms the poles *NS* of the electromagnet are near together and may both act to attract a strip of iron *a*, called the *armature.*

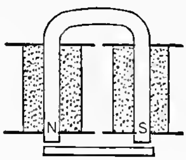

Fig. 300.

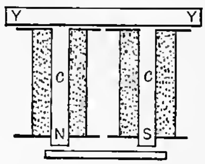

Fig. 301.

512. Retention of magnetism; permanent magnets. — An iron rod retains much of its magnetism when it is removed from a magnetic field in which it has been magnetized; or, in case of an electromagnet, when the magnetizing current is reduced to zero. Long bars retain a greater portion of their magnetism than short bars, because of the fact that in short bars the poles of the magnet are closer together and produce of themselves

a strong demagnetizing field along the bar. The magnetism thus left in a bar of iron or in an electromagnet is called *residual magnetism.* Long bars of annealed wrought iron may retain in this way as much as ninety per cent of their magnetism, but a very weak demagnetizing field or a very slight mechanical shock is sufficient to cause such a bar to lose practically all of its residual magnetism. Cast iron, hard drawn iron wire, and mild steel retain a smaller portion of their magnetism, but with greater persistence, and hardened steel bars retain a portion of their magnetism very persistently, even when roughly handled. Magnetized bars of hardened steel are called *permanent magnets.* The more persistently a sample of iron retains its magnetism, the greater the intensity of magnetic field needed to magnetize it. Thus hardened steel bars are best magnetized by placing them between the poles of a strong electromagnet, or by placing them inside of a large coil of wire, through which a strong current is sent.

Remark.—The region surrounding a magnet is a magnetic field, and capable of magnetizing any piece of iron in the neighborhood. A piece of iron is always magnetized by an adjacent magnet in such a way as to be *attracted* by the magnet. A strong magnet at a distance from a weak one may repel it, but upon being brought nearer, the weak magnet will have its magnetism reversed by the magnetizing action of the intense field due to the strong magnet, and the two magnets will then attract each other. If the weak magnet is then removed to a distance from the strong magnet, it may, if the action of the strong magnet has not been overpowering, regain most of its original magnetism and be repelled as before. Small changes in the magnetic state of a bar are much like elastic distortions, and the bar recovers from them when the cause is removed. This is particularly the case with hardened steel.

513. Intensity of magnetization.—Let m be the strength of the magnetic pole at the end of a long magnetized iron rod

of sectional area q. Then the ratio $\frac{m}{q}$ is called the *intensity of magnetization* I of the rod. That is,

$$I = \frac{m}{q}. \tag{277}$$

514. Magnetic flux outward from a magnetic pole. — Imagine a spherical surface of radius r to be constructed about a magnetic pole of strength m at its center. The field intensity at the spherical surface is $f = \frac{m}{r^2}$, and the product of this field intensity by the area $4\pi r^2$ of the spherical surface gives, as the total magnetic flux outward from the pole,

$$N = 4\pi m. \tag{278}$$

This flux is inwards towards a south pole, and it is conceived to pass through the iron of the magnet from the south pole to the north pole. There is thus a magnetic flux $4\pi m$ or $4\pi Iq$ through an iron rod on account of its magnetized condition. The total magnetic flux through the rod is ordinarily greater than this, as will be shown in the following article.

515. Case of a long rod in a uniform field. — Consider a long iron rod, of sectional area q, placed in a long solenoid* of wire through which a current is sent. The magnetic field surrounding the iron inside this coil is superposed of two distinct parts.

The first of these (a) is the uniform field in the solenoid due to the current. The lines of force of this field are shown by the arrows in Fig. 302. Let $\boldsymbol{H}$ be the intensity of this field. The magnetic flux $\boldsymbol{H}q$, or the $\boldsymbol{H}q$ "lines of force" which come up to the end of the bar, must be thought of as passing on through it. The other part (b) is the magnetic field, due to the poles of the rod, which diverges from the north pole, and converges

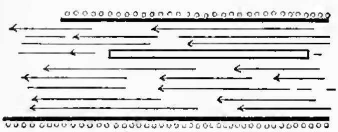

Fig. 302.

* A name given to a cylindrical coil.

upon the south pole of the bar. The lines of this field are not indicated in the figure.

The magnetic flux $4\pi m$ (see previous article), which, because of this field (b) converges upon the south pole, is to be thought of as passing through the bar to the north pole, whence it passes out. Therefore the total magnetic flux through the rod is

$$N = 4\pi m + \boldsymbol{H}q. \tag{279}$$

516. Induction in iron. — The ratio $\frac{N}{q}$ of the total magnetic flux through an iron rod to the sectional area of the rod is called the *induction* $\boldsymbol{B}$ in the rod. That is,

$$\boldsymbol{B} = \frac{N}{q}. \tag{280}$$

From the equation $N = 4\pi m + \boldsymbol{H}q$, using $I = \frac{m}{q}$ (277), and $\boldsymbol{B} = \frac{N}{q}$ (280), we have

$$\boldsymbol{B} = 4\pi I + \boldsymbol{H}. \tag{281}$$

517. Proposition. — The induction $\boldsymbol{B}$ in iron is equal to the intensity of the magnetic field which would exist in a thin transverse slit in the iron.

Proof. — Consider a narrow slit AB, Fig. 303, across a magnetized rod. The magnetic flux through the rod is $\boldsymbol{B}q$, from equation (280), and the magnetic flux across the slit is fq, where f is the field intensity in the slit. The slit being so narrow as not to disturb the flow of magnetic flux, the whole flux through the rod must cross the slit; so that $\boldsymbol{B}q = fq$ or $\boldsymbol{B} = f$.

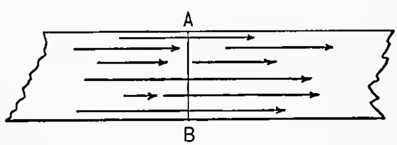

Fig. 303.

518. Magnetizing force in iron. — The magnetizing force $\boldsymbol{H}$ at a point in iron is defined as the field intensity which would be produced at that point in air by all the existing magnetic

poles and electric currents in the neighborhood. This is the field intensity which would exist in a very narrow longitudinal slit in the iron at the point.

The iron rod of which the magnetization is considered in Arts. 515 and 516 is supposed to be long and slim, so that the magnetic field produced along the rod by the poles at its ends may be negligible in comparison with the uniform field due to the long coil. Equations (279) and (281) are entirely general, however, provided $\boldsymbol{H}$ is the actual *magnetizing force* in the iron.

519. Magnetic susceptibility. — Let I be the intensity of magnetization of a sample of iron which has been freshly magnetized from a neutral condition by being subjected to a magnetizing force (field) $\boldsymbol{H}$. The ratio $\frac{I}{\boldsymbol{H}}$ is called the *magnetic susceptibility* k of the sample of iron. That is,

$$I = k\boldsymbol{H}. \tag{282}$$

For all kinds of iron the value of k increases slightly at first and then falls off indefinitely as I increases. Its value varies greatly with the sample of iron.

520. Magnetic permeability. — Let $\boldsymbol{B}$ be the induction in a sample of iron which has been freshly magnetized from a neutral condition by being subjected to a magnetizing force $\boldsymbol{H}$. The ratio $\frac{\boldsymbol{B}}{\boldsymbol{H}}$ is called the *magnetic permeability* μ of the sample of iron. That is,

$$\boldsymbol{B} = \mu\boldsymbol{H}. \tag{283}$$

For all kinds of iron the value of μ increases slightly at first and then falls off, approaching the value unity, as B increases. Its value varies greatly with the sample of iron.

Remark. — From the equations

$$\boldsymbol{B} = 4\pi I + \boldsymbol{H}\ (281),\quad I = k\boldsymbol{H}\ (282),\ \text{and}\ \boldsymbol{B} = \mu\boldsymbol{H}\ (283),$$

we have
$$\mu \boldsymbol{H} = 4\pi k \boldsymbol{H} + \boldsymbol{H},$$
or
$$\mu = 4\pi k + 1. \tag{284}$$

521. The magnetic circuit. — In practice it often occurs that the magnetic flux through a complete circuit of iron is to be considered; for example, in the magnetic circuit of a dynamo and in the iron cores of transformers. Consider a complete circuit formed of an iron rod of uniform sectional area q and of length l. Let a magnetizing force $\boldsymbol{H}$ act in the direction of the rod at each point. Then $N = \boldsymbol{B}q$, and writing $\mu\boldsymbol{H}$ for $\boldsymbol{B}$, we have
$$N = \mu \boldsymbol{H} q. \tag{a}$$

This equation may be written
$$N = \frac{l\boldsymbol{H}}{\frac{1}{\mu}\frac{l}{q}}. \tag{b}$$

Now, $l\boldsymbol{H}$ is the *magnetomotive force*, m. m. f., around the circuit, and if we write
$$MR \equiv \frac{1}{\mu}\frac{l}{q}, \tag{285}$$
we have
$$N = \frac{\text{m. m. f.}}{MR}. \tag{286}$$

The quantity $MR \equiv \frac{1}{\mu}\frac{l}{q}$ is called the *magnetic reluctance* of the circuit, and $\frac{1}{\mu}$ is sometimes called the *specific magnetic reluctance* of the iron. In the derivation of equation (286) the iron circuit is supposed to be of the same sectional area in every part, and all of the same quality of iron. For many practical purposes, however, this equation may be applied to magnetic circuits, such as the magnetic circuit of a dynamo, which are partly of wrought iron, partly of cast iron, and partly of air and copper, and of which the sectional area varies greatly.

522. Specification of magnetic quality of iron. — In the design of ordinary electromagnets it is necessary to know the magnetic quality of the iron which is to be used. For this purpose it is sufficient to know, for a sample of the iron, a series of corresponding values of I and H or of B and H. The latter quantities are in most cases more convenient. The following tables give series of corresponding values of B and H for steel, for gray cast iron, and for annealed refined wrought iron.

TABLE.

MAGNETIC PROPERTIES OF IRON AND STEEL.

Wrought Iron. (Hopkinson.)			Cast Iron. (M. E. Thompson.)*			Cast Steel (average). (M. E. Thompson.)		
H.	B.	μ.	H.	B.	μ.	H.	B.	μ.
10	12400	1240.0	10	5000	500.0	10	9800	980.0
20	14330	716.5	20	6600	330.0	20	12450	622.5
30	15100	503.3	30	7400	246.6	30	13500	450.0
40	15550	388.8	40	7800	195.0	40	14200	355.0
50	15950	319.0	50	8450	169.0	50	14700	294.0
60	16280	271.3	60	8800	146.6	60	15100	251.6
70	16500	235.6	70	9200	131.4	70	15330	219.0

The corresponding curves in Fig. 304 will aid in the comparison of the magnetic properties of these three kinds of iron. It will be seen that the values of B for a given magnetizing force H are slightly greater in wrought iron than in the kinds of steel tested; also that these values are nearly twice as great as the corresponding values of B for cast iron. The sharper elbow of the curve is characteristic of wrought iron, and the more gradual trend is characteristic of cast iron. Steel, however, may be made to give almost any desired curve at will by varying its temper and composition.

* See M. E. Thompson, P. H. Knight, and G. W. Bacon; Trans. of American Inst. of Electrical Engineers, Vol. IX. (1892).

The permeability μ is likewise a function of the degree of magnetization. As will be seen from the curves in Fig. 305, μ decreases at first rapidly and then more slowly with increasing values of $\boldsymbol{H}$.

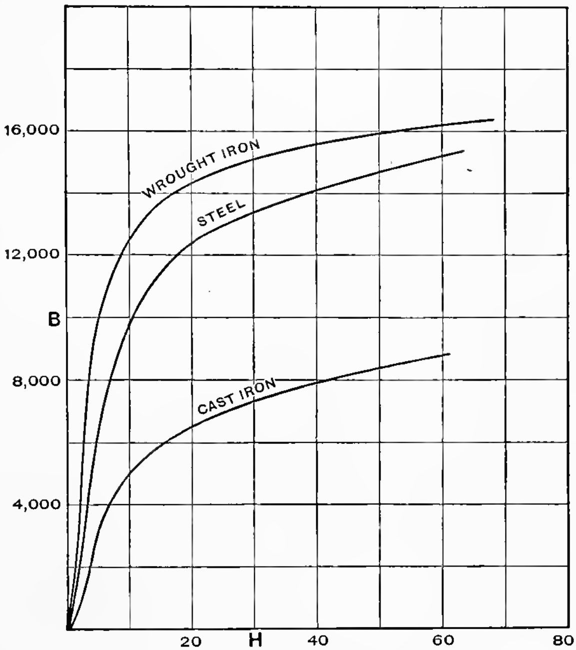

Fig. 304.

523. Work required to magnetize iron. — *Proposition.* The work W done in magnetizing V c.c. of iron is

$$W = \frac{V}{4\pi}\int \boldsymbol{H} d\boldsymbol{B}. \tag{287}$$

Proof. — Consider a long rod of sectional area q, placed in a long coil having n turns of wire per centimeter. The work done in magnetizing this rod is done by forcing current through the coil in opposition to the counter e. m. f. induced in the coil by the increasing magnetic flux through the rod, and the work

thus spent in a section of the coil of length d, having nd turns, is equal to the work used in magnetizing a length d or volume dq of the rod.

The counter e. m. f. in the section of the coil is

$$nd\frac{dN}{dt},$$

where, as in previous articles, N is the magnetic flux through the rod. The rate at which work is done in magnetizing the

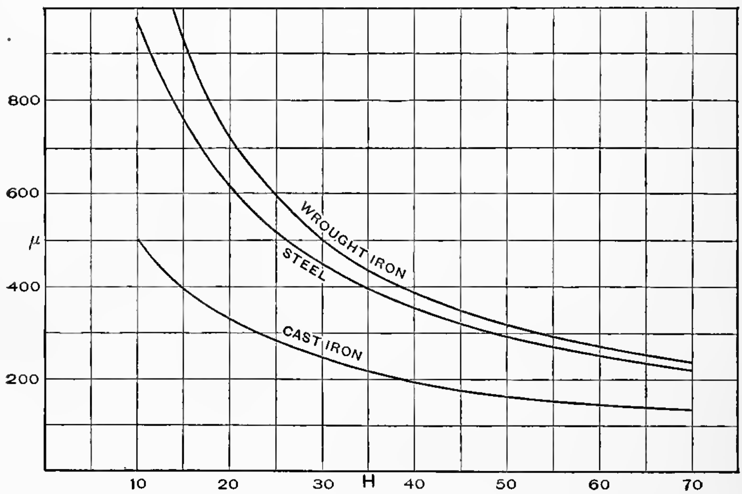

Fig. 305.

length d of the rod, as N is made to increase by increasing i, is

$$\frac{dw}{dt} = ndi\frac{dN}{dt}.$$

Therefore

$$\Delta W = ndi\Delta N \qquad (a)$$

is the work done in magnetizing the rod while the magnetic flux through it is increasing by the amount ΔN.

Now $N = \boldsymbol{B}q$ or $\Delta N = q\Delta\boldsymbol{B}$. Further, from Art. 509, the

magnetizing field in a long solenoid is $\boldsymbol{H} = 4\pi ni$ or $ni = \frac{1}{4\pi}\boldsymbol{H}$. Therefore writing in (a) $q\Delta\boldsymbol{B}$ for ΔN, $\frac{1}{4\pi}\boldsymbol{H}$ for ni, and V for qd, which is the volume of the portion of the rod under consideration, we have

$$\Delta W = \frac{V}{4\pi}\boldsymbol{H}\Delta\boldsymbol{B}. \qquad (b)$$

From this expression, equation (287) results directly.

Remark. — In magnetizing a short rod of iron, more work is required than is given by equation (287); this additional work goes to establish the magnetic field in the neighborhood due to the free magnetic poles of the rod. Equation (287) expresses the work which is spent *within* the iron.

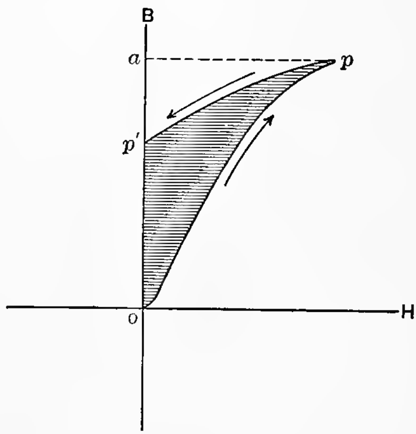

Fig. 306.

524. Graphical representation of work; curve for *B* and *H*. — Let the curve opp' (Fig. 306) be drawn so that the co-ordinates of each point of the curve represent to scale corresponding values of $\boldsymbol{B}$ and $\boldsymbol{H}$ for a given sample of iron when the magnetizing field at first increases from zero to a given value ap and then drops to zero, at which value of $\boldsymbol{H}$ the induction $\boldsymbol{B}$ in the iron retains a value op' (residual magnetism). The abscissas x represent magnetizing field, so that

$$\boldsymbol{H} = ax, \qquad (a)$$

and similarly

$$\boldsymbol{B} = by, \qquad (b)$$

or $d\boldsymbol{B} = b\,dy$. Substituting these values of $\boldsymbol{H}$ and $d\boldsymbol{B}$ in equation (287), we have

$$W = \frac{abV}{4\pi}\int x\,dy. \qquad (288)$$

Now $\int xdy$ is the area between the curve and the Y axis. Therefore the work done in magnetizing the iron from o to p is $\frac{ab}{4\pi}V$ times the area apo, and the work recovered in allowing the iron to demagnetize to p' is $\frac{ab}{4\pi}V$ times the area app'. Therefore the work lost is $\frac{ab}{4\pi}V$ times the shaded area opp'.

525. Hysteresis. — The curve, as in Figs. 304, 306, which represents the values of $\boldsymbol{B}$ and $\boldsymbol{H}$ during the magnetization of iron, does not coincide with the curve which represents the values of $\boldsymbol{B}$ and $\boldsymbol{H}$ during the demagnetization of the iron (see Fig. 306). The property of iron, by virtue of which the deviation of the curves of magnetization and demagnetization occurs, is called *Hysteresis*. On account of hysteresis the work required to magnetize iron is greater than the work regained upon demagnetization, as explained in the previous article. This lost energy appears as heat in the iron. The loss of energy due to hysteresis is distinct when the iron is carried through a complete *cycle* of magnetic changes so as to be brought again into its initial condition, for in this case *all* of the work spent on the iron is hysteresis loss. Thus if an iron rod be repeatedly magnetized, demagnetized, and remagnetized in the opposite sense to the same value of $\boldsymbol{B}$, a curve such as $pap'b$ (Fig. 307) will be obtained for $\boldsymbol{B}$ and $\boldsymbol{H}$, the branch pap' being for increasing values of $\boldsymbol{B}$ and $\boldsymbol{H}$, and the branch $pb'p$ being for decreasing values of $\boldsymbol{B}$ and $\boldsymbol{H}$, as shown by the arrows. The total value of $\int xdy$ (equation (288)) for such a magnetic cycle is the area inclosed in the loop $pap'b$.

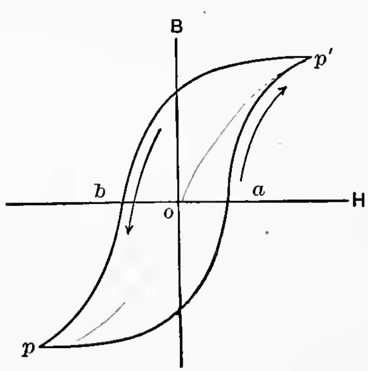

Fig. 307.

When iron is repeatedly magnetized and demagnetized

between any limiting values of $\boldsymbol{B}$, a curve very similar to Fig. 307 is obtained.

Steinmetz has found that the energy W lost in iron per cycle can be expressed with sufficient accuracy for practical purposes by the formula

$$W = nV\boldsymbol{B}^{1.6}, \tag{289}$$

in which V is the volume of the iron, $\pm\boldsymbol{B}$ are the limits of the cycle, and n is a constant which varies with the sample of iron. For annealed refined wrought iron $n = .002$.

Remark. — The values of $\boldsymbol{B}$, μ, and $\boldsymbol{H}$ tabulated in Art. 522 refer to the first magnetization of iron from the neutral condition by an increasing magnetizing field.

The following table gives the hysteresis loss W, in ergs per c.c. of iron per cycle for various ranges of $\boldsymbol{B}$ for annealed refined wrought iron.

TABLE.

Hysteresis Loss per Cycle in Annealed Wrought Iron.

$W\left(\frac{\text{ergs}}{\text{c.c.}}\right)$ (after Swinburne.)	$\pm\boldsymbol{B}$
650	2500
1600	5000
3200	7500
5000	10000
7200	12500
9600	15000

Hammering, or stretching the iron beyond its elastic limits, greatly increases the hysteresis loss for a given range of $\boldsymbol{B}$, and in cast iron and hardened steel the hysteresis loss is vastly greater than in annealed wrought iron. Hysteresis losses affect very materially the efficiency of transformers, the iron cores of which are carried through from 40 to 150 magnetic cycles per second. For this reason the best annealed refined wrought iron is used for transformer cores and for the armatures of dynamos.

526. Magnetic saturation. — The curve (Fig. 308) represents the intensities of magnetization I of a sample of iron for increasing magnetizing force. It will be seen from the diagram that I approaches a definite limiting value as H increases. The iron is said to be *saturated* when it has nearly reached this limiting intensity of magnetization. For soft wrought iron this limiting value of I is 1730, for mild steel it is 1600, for annealed nickel it is 540, and for cobalt it is 1310 (Ewing).

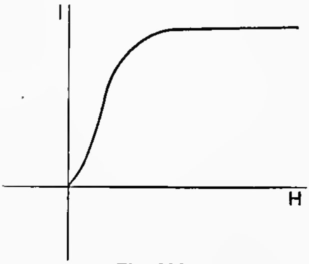

Fig. 308.

527. Molecular theory of the magnetism of iron. — When a steel magnet is broken in pieces each part is found to be a complete magnet with a north pole and a south pole. Many of the facts concerning the magnetism of iron are clearly represented if we think of the ultimate particles or molecules of the iron as magnets. When placed in a strong magnetic field, these molecular magnets are all turned with their north poles in the same direction, and the iron is *magnetized*. When removed from the field the molecular magnets, by their mutual attraction, become disarranged, and the iron is *demagnetized*. Iron acts very much as if the movement of these molecular magnets were opposed by a frictional resistance. In annealed iron this frictional resistance is comparatively slight, in hard drawn iron wire, cast iron, and mild steel it is greater, and in hardened steel it is very great. Mechanical vibration and rise of temperature both act as if to decrease this frictional resistance, enabling a given magnetizing field H to produce more intense magnetization, and causing residual magnetization to disappear.

Ewing has shown * that this apparent frictional opposition to the movement of the magnetic molecules may be due to the mutual action of these molecules as magnets. In fact, a group

* See Phil. Mag. (5), Vol. 30, p. 205.

of magnets supported upon jewels and points may be in equilibrium in various configurations. If subjected to the action of an increasing uniform magnetic field, the magnets in such a group are deflected more and more until the configuration becomes unstable, with the result that the group literally falls into another configuration.

528. Variation of magnetic properties of iron with temperature. — As mentioned above, the permeability of iron rises with increase of temperature up to a critical temperature, which in the case of iron lies between 700° and 800°, but the limiting value of I does not vary greatly with temperature. At a temperature of rather less than 800° C. iron loses completely its peculiar magnetic properties, and above that temperature it is not very different from such substances as copper, wood, glass, air, etc., in its magnetic behavior.

529. Paramagnetism. — Cobalt and nickel are very similar to iron in their magnetic properties except that the limiting value of their intensity of magnetization is not so great. A great many other substances, such as manganese, chromium, platinum, many iron compounds, oxygen, etc., show similar properties, but to a lesser degree. Such substances are said to be *paramagnetic* (Faraday).

530. Diamagnetism. — If a rod of iron, or any paramagnetic substance, is suspended, so as to be free to turn, in a magnetic field, it will turn so as to be parallel to the direction of the field. Many other substances, such as bismuth, antimony, zinc, lead, silver, copper, etc., when suspended in the form of a small bar in a magnetic field, arrange themselves at right angles thereto. Such substances are said to be *diamagnetic* (Faraday). The magnetic permeability, $\frac{B}{H}$, of such substances is less than unity. The diamagnetic property of bismuth is easily shown by suspending a small bar of it between the poles of a strong electromagnet.

Weber has explained diamagnetism as follows: A bar of conducting material has currents induced in it when it is suddenly brought near to a magnet. These currents circulating in the bar make it behave as a magnet, just as a coil of wire carrying a current behaves as a magnet, and this fictitious magnetization is always in such a direction that the bar is repelled by the magnet. Owing to electrical resistance, these induced currents quickly die away, but Weber assumed that, in diamagnetic substances, *molecular currents* were thus induced which do not die away but persist, thus explaining the diamagnetic effect. Thus, if a massive plate of copper is suspended near one pole of an electromagnet it is repelled with very considerable force when the magnet is quickly excited. The force of repulsion due to the genuine diamagnetic property of the copper is very much smaller and not easy to detect.

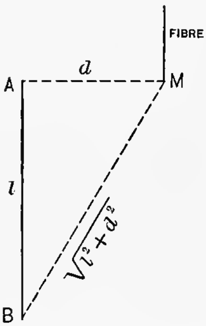

Fig. 309.

531. Ewing's method of testing iron. — A rod AB (Fig. 309) of the iron to be tested, of length l (three or four hundred times its diameter) and sectional area q, is placed vertically in a long magnetizing solenoid having n turns of wire per centimeter. The upper end of the rod A is placed at a distance d, due magnetic east or west, of a suspended magnet M. Let m be the pole strength of A and B. Then the field at M due to A is $\frac{m}{d^2}$, and the horizontal component of the field at M due to B is $-\frac{m}{l^2+d^2}\frac{d}{(l^2+d^2)^{\frac{1}{2}}}$. The total field at M due to the rod is the sum of these two, and this total field will be at right angles to the earth's horizontal field H_1, at M, so that

$$\tan\phi = \frac{\frac{m}{d^2} - \frac{md}{(l^2+d^2)^{\frac{3}{2}}}}{H_1}. \qquad (a)$$

In this equation ϕ is the angle of deflection of the magnet M produced by the rod. If ϕ, d, and l are observed and H_1 is known, m may be calculated from (a) whence $I = \frac{m}{q}$ is known. The magnetizing field H acting on the rod is

$$H = 4\pi ni, \qquad (276\ bis)$$

in which i is the observed magnetizing current in the solenoid. B and μ are then calculated from equations (281) and (283). In this way a series of corresponding values of B, μ, and H may be determined. The most troublesome errors in this method are as follows:

1. The magnetizing field is somewhat less than $4\pi ni$ because of the demagnetizing field due to the free poles A and B. To render this error small the rod must be made very long in comparison to its sectional area. This error may be allowed for if the test piece is in the form of an elongated ellipsoid.

2. The current in the solenoid acts directly upon the magnet M and produces some deflection, whereas equation (a) assumes the deflection to be due entirely to the field which emanates from the free poles of the rod. This error is easily allowed for by observing the deflection of M by a current in the solenoid when the rod is removed therefrom.

3. The poles A and B are not concentrated but are distributed over considerable portions of the ends of the rod. This source of error is in part provided against by placing the end A of the rod slightly higher than M, and taking l in equation (a) rather less than the actual length of the rod.

532. The ballistic method of testing iron (Rowland). — A ring of the iron to be tested (A, Fig. 310), of section area q and peripheral length l, is wound uniformly with Z turns of wire through which the magnetizing current i is sent. Another coil of n turns, not shown in the figure, is wound

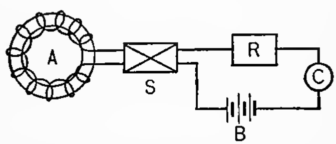

Fig. 310.

upon the ring and connected with a ballistic galvanometer. The

magnetizing current, furnished by a battery B, flows through an ammeter C, a rheostat R, so arranged as to enable the observer to produce quick changes in the current, and a reversing switch S.

The m. m. f. around the iron ring is $4\pi Zi$ by equation (275), whence from equation (273) we have the magnetizing field.

$$H = \frac{4\pi Zi}{l}. \tag{a}$$

If i be suddenly changed by a measured amount Δi, we have

$$\Delta H = \frac{4\pi Z}{l}\Delta i \tag{b}$$

and the corresponding change in B is

$$\Delta B = \frac{kra}{nq}, \tag{c}$$

in which k is the reduction factor of the ballistic galvanometer, r is the total resistance of the ballistic galvanometer circuit, and a is the observed throw of the ballistic needle produced by the change in B.

Proof of equation (*c*).—The changing flux through the iron induces an e. m. f. $n\frac{dN}{dt}$ in the n-turns of wire in circuit with the ballistic galvanometer. This e. m. f. produces a current $\frac{dQ}{dt}$ in that circuit, which by Ohm's law is equal to e. m. f. ÷ resistance. That is, $\frac{dQ}{dt} = \frac{n}{r}\frac{dN}{dt}$ or $Q = \frac{n}{r}\Delta N$, where Q is the charge which circulates through the ballistic galvanometer while the magnetic flux through the iron changes by the amount ΔN, or while the induction in the iron changes by the amount $\Delta B = \frac{\Delta N}{q}$. From the equation for the ballistic galvanometer (see Art. 386) we have $Q = k \cdot a$; therefore, writing $k \cdot a$ for Q and $q \cdot B$ for ΔN in $\Delta Q = \frac{n}{r}\Delta N$, we have $\Delta B = \frac{kra}{nq}$.

The curve of B and H is plotted from a series of observed

values of ΔH and ΔB as follows: Beginning at any point, we lay off ΔH and ΔB to scale. This determines the next point of the curve. From this point we lay off the next observed values of ΔH and ΔB, thus locating the next point of the curve, and so on. The whole curve is thus drawn and the axes of B and H, if it is desired to indicate them, can be drawn through the center of the figure parallel to ΔB and ΔH respectively.

The most troublesome errors in this method are the following:

1. The magnetizing field, equation (a), is greater in the inner portion of the ring where l is smaller. This lack of uniformity in the magnetizing field introduces complications not considered in the equations (b) and (c), and these equations will therefore, in general, give erroneous results. These errors are in great part obviated by using a ring of such dimensions that l differs but little in various parts of it, and by using a mean value for l in equation (a).

2. Equation (c) takes account only of changes in $\boldsymbol{B}$ which occur promptly, during an interval of time which is but a fraction of the time of vibration of the ballistic needle.

In some kinds of iron (very soft wrought iron) a quick change in $\boldsymbol{H}$ produces a prompt change in $\boldsymbol{B}$, followed by a sluggish change which continues for a few seconds. Equation (c) in this case leads to slightly erroneous results.

Any slow change in the magnetizing current, due for example to heating of the wires in circuit or to the polarization of the battery, produces a corresponding slow change in $\boldsymbol{H}$ and $\boldsymbol{B}$, in which case also the use of equation (c) leads to erroneous results.

CHAPTER XII.

INDUCED ELECTROMOTIVE FORCE; MUTUAL AND SELF INDUCTION.

533. Induced e. m. f.—Consider a circuit (Fig. 311) consisting of two parallel straight pieces of wire AB, AB, distant l from each other, connected permanently at AA and bridged at BB by a piece of wire of length l. Let the latter be so arranged that it can be made to slide sidewise on AB, AB, as on a pair of rails at velocity v. Let i be the electric current flowing in the circuit in the direction shown by the curved arrows, and imagine the whole arrangement to be placed in a uniform magnetic field, of intensity f, at right angles to the plane of the paper, and directed towards the reader. Then by Ampere's law (Art. 353), a force

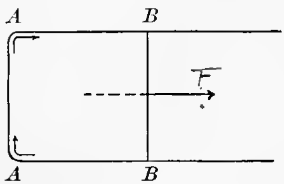

Fig. 311.

$$F = ilf \tag{a}$$

will urge BB sidewise, as shown by the arrow F.

Let BB be pulled sidewise at velocity v by some agent. This agent will do more work in moving BB than if the opposing force F were not acting. The extra power P expended by this agent on account of the opposing force F is

$$P = Fv; \tag{b}$$

whence, using the value of F from (a) we have

$$P = ilfv. \tag{c}$$

Now it is a result of experiment that all *work expended in moving a wire in opposition to forces due to the action of a mag-*

netic field upon the current, goes to help maintain the current. It follows, therefore, that P in equation (c) is the rate at which the agent which is moving the wire does work in maintaining the current. It has already been shown, however, in Chapter I. of this volume (see Art. 378, equation (219)), that $P = Ei$. The quantity lfv is therefore an electromotive force E, produced in the moving wire BB by the magnetic field f. That is,

$$E = lfv. \tag{290}$$

The e. m. f. E is said to be *induced* in the moving wire.

During a time interval Δt the bar BB (Fig. 311) moves a distance $v \cdot \Delta t$, and $l \cdot v\Delta t$ is the increment of the area of the circuit AB, AB during this time. Now the product of the whole area of AB, AB into the field intensity f gives the magnetic flux N through the circuit, and the product of the increment of area, $l \cdot v\Delta t$ into f, gives the increment of this flux. We have, therefore, $\Delta N = lfv \cdot \Delta t$, or $\frac{\Delta N}{\Delta t} = flv$, and from equation (290) we have

$$E = -\frac{dN}{dt}. \tag{291}$$

This equation is susceptible of rigorous proof for any moving circuit in any magnetic field, using Ampere's law. It is also found to hold for stationary circuits in magnetic fields of changing intensity. Therefore *the induced e. m. f. in any circuit whatever is equal to the rate of change of the magnetic flux through the opening of the circuit.* The negative sign is chosen for reasons explained in Arts. 537 and 538.

534. The direct current dynamo. — Consider an iron ring R (Fig. 312), which is wound with C turns of insulated wire, as shown, the ends of the wire being spliced and soldered so that the winding is endless. Let this iron ring be placed between the north and south poles of a strong magnet, as shown in Fig. 313, and rotated n revolutions per second, as indicated by the curved arrow. Magnetic flux will emanate from the N-pole of the

magnet, into the iron of the ring, will be divided equally between the upper and lower limbs of the ring, and will flow out of the ring into the S-pole, as indicated by the dotted lines. Let N be the total magnetic flux crossing from N to S through the ring.

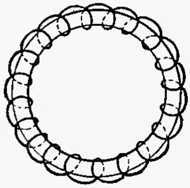

Fig. 312.

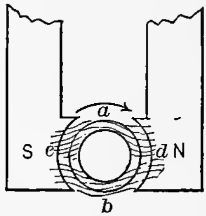

Fig. 313.

Consider a given turn of wire on the ring. When this turn is at a, the magnetic flux through it is $\frac{N}{2}$. After one-half a revolution, or $\frac{1}{2n}$ seconds, this turn will be at b, and the flux through it will be $-\frac{N}{2}$. The flux is now to be considered negative because the turn of wire presents its reverse side towards the N-pole. The total change of flux through this turn during this half revolution is therefore N, which, divided by the elapsed time, $\frac{1}{2n}$ seconds, gives $2nN$ as the average rate of change of flux through the turn of wire while it is passing from a to b. Therefore, by equation (291), $2nN$ is the average induced e. m. f. in the given turn of wire while it is moving from a to b. There are $\frac{C}{2}$ turns of wire on the half of the ring between ab, so that $\frac{C}{2} \times 2nN$ is the total e. m. f. E between a and b. That is,

$$E = NCn. \tag{292}$$

This e. m. f. cannot produce a current in the endless wire which is wound on the ring, for the reason that exactly equal and opposite e. m. f.'s are induced in the wire on the two sides of the ring, as shown schematically in Fig. 314, in which figure the circle represents the endless wire which is wound on the

ring. A current can be taken from the winding of the ring through an outside circuit *l* (Fig. 315), by keeping the ends of this circuit in metallic contact with the windings on the ring at *a* and *b*. For this purpose the insulation may be removed from the outer portions of the windings on the ring, or from every second turn, or every third turn, etc., and metallic springs *ss* (Fig. 316) may be arranged to rub at *a* and *b* as the ring rotates. In practice, wires are soldered to the various turns, or to every second turn, or every third turn, etc., of the ring winding, and led down to insulated copper bars fixed near the axis of rotation. Sliding contact is then maintained with these bars, instead of with the turns of wire at *a* and *b* directly. (See Fig. 316.)

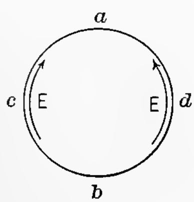

Fig. 314.

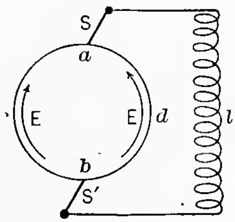

Fig. 315.

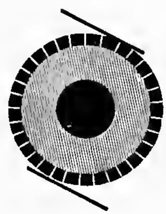
Fig. 316.

A machine for obtaining an electric current by the method just described is called a *direct current dynamo*. The rotating ring is called the *armature*, and the magnet *NS* (Fig. 313) is called the *field magnet*. The insulated copper bars which are connected to the various turns constitutes what is called the *commutator*, and the metal or carbon strips which rub on the commutator are called the *brushes*. The field magnet is ordinarily an electromagnet, which is excited by the current generated by the dynamo itself.

Equation (292) is the fundamental equation of the dynamo. The type of armature here described is called the *ring* armature. It is essentially identical to the *drum* armature.

535. The alternating current dynamo. — This dynamo consists essentially of a coil of wire, which is moved near a

magnet in such a way that the magnetic flux goes through the coil first in one direction and then in the other. This induces an e. m. f. in the coil in one direction while the flux is increasing, and in the other direction while the flux is decreasing. This *alternating e. m. f.* produces an *alternating current* in the coil, and in any outside circuit which is kept in continuous connection with the ends of the wire of the coil.

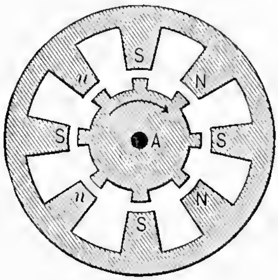

Fig. 317.

Such a device is called an alternating dynamo, or simply an *alternator.* A common type of the alternator consists of a multipolar electromagnet (*the field magnet*), of which the poles project radially inwards towards the passing teeth of a rotating toothed iron wheel *A* (*the armature*), as shown in Fig. 317. On the armature shaft, at one end of the armature, are fixed two insulated metal rings (*collecting rings*), upon which metal springs (*brushes*) rub, keeping continuous contact with the terminals of an external circuit (Fig. 318). The ends of the armature wire are soldered to the respective collecting rings, and the wire is wound around the armature teeth, in opposite directions around adjacent teeth. The field magnet of the alternating dynamo is excited by a continuous current from some independent source.

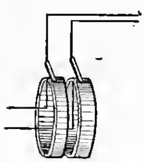
Fig. 318.

536. Eddy currents; lamination. — When a massive piece of iron is magnetized or demagnetized, the changing magnetic flux through the central portions induces e. m. f.'s around the outer portions. These e. m. f.'s produce what are called *eddy currents* or *Foucault currents* in the iron. Those iron parts of electrical machinery which are subject to rapid and frequent changes of magnetism are therefore built up of thin sheet iron or of iron wire, so as to leave the metal continuous in the direc-

tion of magnetization and discontinuous at right angles to this direction. Thus the flow of eddy currents is greatly diminished. The iron parts of all dynamo armatures and all transformer cores are *laminated* in this way. Eddy currents are also produced in any massive piece of metal which is moved in a magnetic field. For this reason the large copper wires on dynamo armatures are often made of a bundle of fine wires.

537. Positive directions around and through a circuit. — The positive direction around a circuit being chosen arbitrarily, the positive direction through the loop formed by the circuit is taken as the direction in which a right-handed screw would move through the circuit if turned in the direction which has been chosen as the positive direction around the circuit. Electric currents and electromotive forces are considered positive when they are in the positive direction around a circuit. A magnetic flux through a circuit is considered positive when a north-pointing magnetic pole would be drawn in the positive direction through the circuit by the magnetic field producing the flux.

538. In the present discussion we shall make use of the following principles:

(*a*) The induced e. m. f. in a circuit is equal to the rate of change of the flux through the loop formed by the circuit. That is,

$$E = -\frac{dN}{dt}. \qquad (291\ bis)$$

The negative sign is chosen for the reason that an increasing positive flux produces a left-handed e. m. f. around the circuit.

(*b*) The work ΔW done in keeping a current i in a circuit constant while the magnetic flux through the opening of the circuit is changed by an amount ΔN is

$$\Delta W = i\Delta N. \qquad (268\ bis)$$

(See Art. 504.)

(*c*) The intensity f of the magnetic field at a given point near a given coil of wire is

$$f = Gi, \tag{203 bis}$$

in which i is the current in the coil, and G is a constant for the given point and coil. The direction of f at the given point remains unchanged as the current is increased.

(*d*) The magnetic field is a seat of kinetic energy. The kinetic *energy in ergs per cubic centimeter* in the neighborhood of a point in air is

$$W = \frac{1}{8\pi} f^2, \tag{198 bis}$$

in which f is the intensity of the magnetic field at the point.

539. Inductance of a coil. — When a current is established in a coil, a magnetic field is produced in the neighborhood of the coil. Therefore a certain amount of kinetic energy, viz., the energy of the field, is associated with the current in the coil. The intensity f of the magnetic field at each point is proportional to the current i in the coil by equation (203), and the kinetic energy in each element of volume of the surrounding region is proportional to the square of f at the element. Therefore the kinetic energy in each element of volume is proportional to i^2, and *the total kinetic energy in the surrounding region is proportional to* i^2, so that we may write

$$W = \tfrac{1}{2} Li^2. \tag{293}$$

In this equation, W is the kinetic energy associated with a current i in a given coil, and $(\frac{1}{2} L)$ is the proportionality factor. The quantity L is called the *inductance* of the coil.* It depends upon the size, shape, and number of turns of wire in the coil, and is essentially positive. The coil is supposed to be surrounded by air.

* This term L is sometimes called the coefficient of self-induction.

540. Self-induced electromotive force and magnetic flux due to a current in a coil. — If i in equation (293) varies, then

$$\frac{dW}{dt} = Li\frac{di}{dt}.$$

When $\frac{dW}{dt}$ is positive, power is expended on the coil; for $\frac{dW}{dt}$ is the rate at which the kinetic energy of the current is increasing. When power is thus expended on a coil (aside from the quantity i^2R), this power is equal to the product of an e. m. f. E into the current i. We have then

$$\frac{dW}{dt} = Li\frac{di}{dt} = Ei, \tag{294}$$

in which $E = L\frac{di}{dt}$ is that portion, of the total e. m. f. forcing current through the coil, which is used to make the current increase at the rate $\frac{di}{dt}$. We conceive an e. m. f. e to be induced in the coil by the increasing current, and this e. m. f. is equal and opposite to E, so that

$$e = -L\frac{di}{dt}. \tag{295}$$

The e. m. f. e is called *a self-induced e. m. f.* and is produced by changing the current.

The self-induced e. m. f. e is due to a changing magnetic flux through the coil; hence, using equation (291) with (295), we have

$$\frac{dN}{dt} = L\frac{di}{dt}.$$

By integration* from $N = 0$ and $i = 0$, we then obtain

$$N = Li, \tag{296}$$

where N is the magnetic flux through the coil, due to the current i in the coil.

* See footnote to Art. 386.

541. The extra current. — Let a constant impressed e. m. f. E begin to act on a circuit at a given instant. The self-induced e. m. f. is $-L\frac{di}{dt}$ from (295), so that the effective e. m. f. is $E-L\frac{di}{dt}$, and the current i is $i=\frac{E-L\frac{di}{dt}}{R}$ by Ohm's law, R being the resistance of the circuit. This expression for i may be written $i=\frac{E}{R}-\frac{L}{R}\frac{di}{dt}$; that is, the actual current may be considered to be superposed of the two currents $\frac{E}{R}$ and $-\frac{L}{R}\frac{di}{dt}$. The latter is called the *extra current*.

542. Circulation of charge due to extra current. — Let $\frac{dQ}{dt}$ be the rate at which charge circulates in the circuit on account of the extra current, then $\frac{dQ}{dt}=-\frac{L}{R}\frac{di}{dt}$. Integrating* from $Q=0$ and $i=0$, we have

$$Q=-\frac{Li}{R}. \tag{297}$$

Q is the total quantity of electricity which circulates in the circuit due to the extra current while the actual current is changing from zero to i; this circulation is evidently counter to i. The charge which circulates when the current changes from i to zero is $\frac{Li}{R}$ in the direction of i.

Remark. — The total circulation of charge in a coil due to the extra current produced by self-induction is zero when the initial and final values of the actual current are equal.

543. (a) **Proposition.** — *The inductance of a large coil of small sectional area is proportional to n^2, n being the number of turns of wire in the coil.*

Proof. — With given current, the magnetic field at each point is proportional to n, so that the energy per unit volume at each

* See footnote to Art. 386.

point and hence also the total energy is proportional to n^2. It follows, from equation (293), L must be proportional to n^2.

Remark. — The *magnetic flux through a coil* is equal to the product of the flux, through a mean turn, into the number of turns. Thus the flux through a large coil having small sectional area or the flux through a coil wound upon an iron core is equal to the product of the number of turns of wire in the coil into the flux through the opening of the coil.

(*b*) **Proposition.** — *The inductance of any coil which is so arranged that the magnetic flux due to each turn flows through each remaining turn is proportional to* n^2.

Proof. — With given current, the magnetic field at each point is proportional to n, the flux through the opening of the coil is therefore proportional to n. This flux must be multiplied by n to give N in equation (296), so that i being given L will vary as N or as n^2.

Remark. — The condition specified in proposition (*b*) is approximately satisfied by a large coil of small sectional area and by a coil wound upon an iron core. In case of a coil wound upon an iron core, N is *not* strictly proportional to i, so that the inductance of such a coil has no perfectly definite value.

544. Mutual induction of two coils. — Consider two coils, a primary and a secondary, in which the currents i_1 and i_{11} are flowing. Let f_1 and f_{11} be the strengths of magnetic field at a point p (Fig. 319) due to i_1 and i_{11} respectively. Then f_1 and f_{11} are proportional to i_1 and i_{11} respectively, and the angle ϕ f_1 and f_{11} is between independent of i_1 and i_{11}. (See Art. 538.) Let f_{111} be the resultant field at p, then

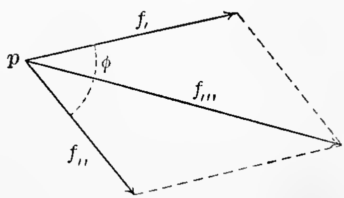

Fig. 319.

$$f_{111}^2 = f_1^2 + f_{11}^2 + 2 f_1 f_{11} \cos \phi.$$

The kinetic energy per unit volume at p consists therefore of three parts proportional respectively to f_1^2, to f_{11}^2, and to $f_1 f_{11}$ or proportional to i_1^2, to i_{11}^2, and to $i_1 i_{11}$ respectively. Therefore the total kinetic energy W associated with the two currents consists of three such parts, so that

$$W = \tfrac{1}{2} L_1 i_1^2 + \tfrac{1}{2} L_{11} i_{11}^2, \text{ i.e. } i_{11}^2 + M i_1 i_{11}. \tag{298}$$

L_1 and L_{11} are the values of the inductance of the respective circuits as defined by equation (293), for equation (298) reduces to (293) when either $i_1 = 0$ or $i_{11} = 0$. The quantity M is called the *mutual inductance* or *coefficient of mutual induction* of the two coils. Its value depends upon the size, shape, number of turns, and relative position of the two coils. It may be either positive or negative.

545. Units of self and mutual induction. — When the currents i_1 and i_{11} in equation (298) are expressed in c. g. s. units and W in ergs, L_1, L_{11}, and M are said to be expressed in c. g. s. units. The physical dimensions of L and M are *length*, consequently the c. g. s. units of L and M are called the *centimeter*.

When W in equation (298) is expressed in joules (10^7 ergs) and currents are expressed in amperes, L and M are expressed in terms of a unit called the *henry*. One henry is equal to 10^9 cm.

546. Production of e. m. f. by mutual induction and magnetic flux due to mutual induction. — Let power be applied to the two coils, Art. 544, so that i_{11} may be kept constant and i_1 be made to change at rate $\frac{di_1}{dt_1}$, and *let it be required to find how this power must be distributed between the two circuits.* Differentiating (298) with respect to time, remembering i_{11} to be constant, we have

$$\frac{dW}{dt} = L_1 i_1 \frac{di_1}{dt} + M i_{11} \frac{di_1}{dt}. \tag{299}$$

Any expression for expended power which involves the current in a circuit as a factor requires the other factor to be a *counter e. m. f.* in the same circuit, and it is of course on this circuit that the power is expended. Therefore power equal to $L_1 i_{11} \frac{di_1}{dt}$ must be expended on the primary coil and $M i_{11} \frac{di_1}{dt}$ must be expended on the secondary in order that i_1 may change and i_{11} remain constant. It follows that $L_1 \frac{di_1}{dt}$ is a counter self-induced e. m. f. in the primary coil and $M \frac{di_1}{dt}$ is a counter e. m. f. e_{11}, induced in the secondary by the changing current in the primary. We have, therefore,

$$e_{11} = -M \frac{di_1}{dt}; \tag{300}$$

and in similar manner we find

$$e_1 = -M \frac{di_{11}}{dt}. \tag{301}$$

The e. m. f. e_{11} in equation (300) is due to a changing magnetic flux N_{11} through the secondary which is produced by the primary current. We may write, therefore, $\frac{dN_{11}}{dt} = M \frac{di_1}{dt}$; whence, integrating from $N_{11} = 0$ and $i' = 0$, we have

$$N_{11} = M i_1. \tag{302}$$

Similarly,

$$N_1 = M i_{11}. \tag{303}$$

The magnetic flux through a coil due to a current in another coil is therefore equal to the product of the current into the mutual inductance of the coils.

547. Circulation of charge due to mutual induction. *Preliminary statement.* — The initial and final values of the current in a coil being equal, the calculation of the circulation of charge in that coil, produced by any action whatever, may be based upon the assumption that the inductance of the coil is zero. (See Art. 542.)

Let it be required to calculate the circulation of charge in a secondary coil when the current in the primary changes. The e. m. f. e_{11}, equation (300), will produce a current in the secondary coil ($L_{11} = 0$) equal to $\frac{e_{11}}{R_{11}}$, where R_{11} is the resistance of the coil; and this current will be equal to the rate of circulation of charge. We have, therefore, $\frac{dQ''}{dt} = -\frac{M}{R_{11}}\frac{di_1}{dt}$. Integrating from $i_1 = 0$ and $Q_{11} = 0$, we have

$$Q_{11} = -\frac{Mi_1}{R_{11}}. \tag{304}$$

Similarly,

$$Q_1 = -\frac{Mi_{11}}{R_1}. \tag{305}$$

The simplest method for the determination of the mutual inductance of two coils is to measure by means of the ballistic galvanometer, the circulation of charge in one coil due to a known change of current in the other.*

548. The force action between two coils. *Preliminary statement.* — The work ΔW expended on a circuit to maintain a current i in that circuit constant while the magnetic flux through the circuit changes by an amount ΔN, is

$$\Delta W = i\Delta N. \tag{i}$$

(See Art. 504.)

Let it be required to find the work expended on two circuits when one of them is displaced by an amount Δx, the other being fixed, the currents i_1 and i_{11} being maintained constant. Let ΔM be the change in M due to the displacement. From equation (298), L_1, L_{11}, i_1, and i_{11} being unchanged, we find that the kinetic energy of the system will increase by an amount

$$a = \Delta M i_1 i_{11}. \tag{ii}$$

Also from equations (302) and (303) we have $\Delta N_1 = i_{11}\Delta M$ and $\Delta N_{11} = i_1 \Delta M$. These are the changes of flux in the re-

* See Elements of Electricity and Magnetism, J. J. Thomson, pp. 434–442, for methods for measuring self and mutual inductance.

spective coils produced by the displacement. Therefore by equation (i) an amount of work

$$b = \Delta M \cdot i_1 i_{11} \tag{iii}$$

must be spent on the primary to keep the current in that circuit from changing, and an amount of work

$$c = \Delta M \cdot i_1 i_{11} \tag{iv}$$

must be spent upon the secondary circuit to keep the current in that circuit constant The total energy expended upon the system is therefore $2\Delta M i_1 i_{11}$ by equations (iii) and (iv), and the increase in the energy of the system is $\Delta M i_1 i_{11}$ by equation (ii); therefore the movement Δx must have been helped by a force X, such that $X\Delta x = \Delta M i_1 i_{11}$, in order that the energy $2\Delta M i_1 i_{11} - \Delta M i_1 i_{11}$ may escape from the system by pushing upon the displaced coil. From $X \cdot \Delta x = \Delta M i_1 i_{11}$ we have

$$X = \frac{dM}{dx} i_1 i_{11}. \tag{306}$$

That is, the force X, with which one coil tends to produce a displacement Δx of the other, is equal to the product of the currents i_1 and i_{11} multiplied by the ratio $\frac{\Delta M}{\Delta x}$, where ΔM is the change in M produced by the displacement. This force tends to increase M when the product $i_1 i_{11}$ is positive. If Δx is an angular displacement about a given axis, then X is a torque acting about the same axis.

549. Production of e. m. f. by motion of one coil. — The work $b = \Delta M i_1 i_{11}$ (Art. 548) expended in the primary, and the work $c = \Delta M i_1 i_{11}$ expended in the secondary, show that there has been *counter e. m. f.'s* in these coils during the motion Δx. Let $\frac{dx}{dt}$ be the velocity at which this displacement is performed, then the coefficient of mutual induction will be changing at a rate $\frac{dM}{dt} = \frac{dM}{dx}\frac{dx}{dt}$ during the motion. Let e_1 be the e. m. f. in the

primary produced by the motion, then $\Delta M i_1 i_{11} = - e_1 i_1 \Delta t$, whence $e_1 = - i_{11}\frac{dM}{dt}$ or, writing $\frac{dM}{dx}\frac{dx}{dt}$ for $\frac{dM}{dt}$, we have

and similarly
$$\left.\begin{aligned} e_1 &= - i_{11}\frac{dM}{dx}\frac{dx}{dt}, \\ e_{11} &= - i_1\frac{dM}{dx}\frac{dx}{dt}. \end{aligned}\right\} \qquad (307)$$

550. (*a*) **Proposition.** — *Two coils connected in series constitute a single coil of which the inductance is* $L_1 + L_2 + 2M$.

This becomes at once apparent if we write $i_1 = i_2$ in equation (298), and compare the resulting equation with (293).

(*b*) **Proposition.** — *The force action between two coils connected in series is proportional to the square of the current.* This may be shown by writing $i_1 = i_{11}$ in equation (306).

(*c*) **Proposition.** — *Two coils so arranged that all the magnetic flux through the opening of the one flows through the opening of the other, give*

$$M = \sqrt{L_1 L_{11}}. \qquad (308)$$

Proof. — Let P be the magnetic flux through the openings of the two coils, due to a current i in the primary, and let S be the magnetic flux through the openings of the coils, due to a current i_{11} in the secondary. Let n_1 and n_{11} be the number of turns of wire in the coils.

Then

$M i_1 = P n_{11}$ (i) (see equation (302)).
$L_1 i_1 = P n_1$ (ii) (see equation (296)).
$M i_{11} = S n_1$ (iii) (see equation (302)).
$L_{11} i_{11} = S n_{11}$ (iv) (see equation (296)).

If we multiply (ii) and (iv) member by member, and (i) and (iii) member by member, and compare the resulting equations, we find $M = \sqrt{L_1 L_{11}}$. Q.E.D.

551. Examples of mutual induction and self induction. (*a*) *Mutual inductance of a large circular coil (primary) and a small coil (secondary) at its center.* — Let r_1 and r_{11} (Fig. 320) be the mean radii, and n_1 and n_{11} the number of turns of wire in the respective coils, and let θ be the angle between the planes of the coils. The magnetic field at the center of the large coil, due to the current i_1 in that coil, is perpendicular to the plane of the coil and equal to $\frac{2\pi n i_1}{r_1}$.

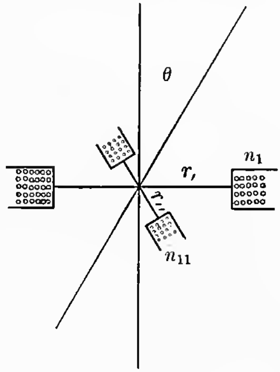

Fig. 320.

The projection, upon the plane of the large coil, of the effective area of the small coil is $\pi r_{11}^2 n_{11} \cos\theta$, therefore the magnetic flux through the small coil due to the current i_1 is $\frac{2\pi n_1 i_1}{r_1} \cdot \pi r_{11}^2 n_{11} \cos\theta$. This flux is equal to Mi_1. (See equation (303).)

We have, therefore,

$$M = \frac{2\pi^2 n_1 n_{11} r_{11}^2 \cos\theta}{r_1}. \tag{309}$$

The substitution of this value of M in equation (306) gives

$$T = \frac{2\pi^2 n_1 n_{11} r_{11}^2 \sin\theta}{r_1} i_1 i_{11}, \tag{310}$$

in which T is the torque tending to decrease θ, and in which i_1 and i_{11} are positive, and $\sin\theta$ is positive.

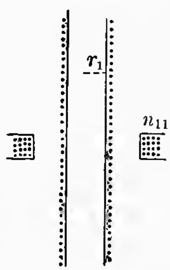

Fig. 321.

(*b*) *Mutual inductance of a long solenoid (primary) and a short coil (secondary) surrounding it.* — Let n_1 be the number of turns of wire per centimeter of the solenoid, and let r_1 (Fig. 321) be its mean radius. The magnetic field within the solenoid will then be $4\pi n_1 i_1$. This, multiplied by the area of the open-

ing of the solenoid (πr_1^2), gives $4\pi^2 n_1 r_1^2 i_1$ as the flux through the opening. This flux also passes through the n_{11} turns of the secondary, so that

$$N_{11} = n_{11} \times 4\pi^2 n_1 r_1^2 i_1;$$

and since $N_{11} = Mi$, i.e. i_1, we have

$$M = 4\pi^2 n_1 n_{11} r_1^2. \tag{311}$$

(*c*) *Inductance per unit length of a long solenoid.* — The flux through the *opening* of the solenoid is $4\pi^2 n r^2 i$, which, multiplied by the number of turns per unit length n, gives the flux through unit length of the coil, which by equation (296) is equal to Li. Therefore

$$L = 4\pi^2 n^2 r^2. \tag{312}$$

This equation may be used for calculating the approximate inductance of a solenoid of any length. For this purpose we multiply $4\pi^2 n^2 r^2$ by the length of the solenoid. This always gives an overestimate of the self inductance, the overestimate being greater the shorter the coil in comparison with its radius.

552. The electrodynamometer. — (*a*) *Weber's form*, which has already been described in Art. 364, consists of a small circular coil suspended at the center of a large circular coil (Fig. 322), provision being made for measuring the torque which acts upon the small coil. The two coils are connected in series, so that equation (310) becomes

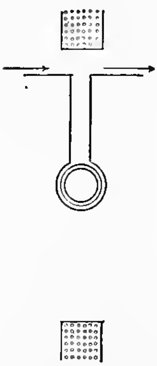
Fig. 322.

$$T = -\frac{2\pi^2 n_1 n_{11} r_{11}^2 \sin\theta}{r_1} i^2. \tag{313}$$

From this i may be calculated when n_1, n_{11}, r_1, r_{11}, are known, and θ and T have been observed. This form of electrodynamometer is sometimes

called the absolute electrodynamometer, since it enables the measurement of current directly in terms of mechanical units. (Compare Art. 364.)

(*b*) *Siemens' form.* — In this type of electrodynamometer the movable and the stationary coils (Fig. 323) are nearly of the same size, so that the computation of the numerical value of M and $\frac{dM}{d\theta}$ is not feasible. The movable coil is always kept in the same position however, so that the ratio $\frac{dM}{d\theta}$ is a constant for a given instrument. The torque acting upon the movable coil is therefore proportional to the square of the current. This torque is balanced by the torsion of a spring, the angle of twist (ϕ) of which is observed, and since T is proportional to ϕ and to i^2, we have

$$i = k\sqrt{\phi}. \tag{314}$$

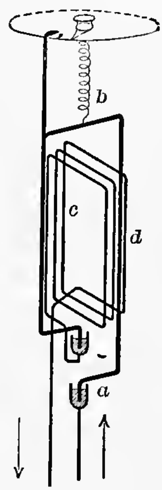

Fig. 323.

The quantity k is called the reduction factor of the instrument. It is determined by noting the value of ϕ when a known current traverses the coils of the electrodynamometer.

Phenomena of Mutual and Self Induction.

553. Spark at break. — When an inductive circuit, that is, a circuit of which the inductance is not zero, is broken, the current very quickly drops to zero. Therefore the rate of decrease $\frac{di}{dt}$ of the current is enormously great and the self-induced e. m. f. $L\frac{di}{dt}$ is very large. The self-induced e. m. f. is in the direction of the current when $\frac{di}{dt}$ is negative, that is, when the current is decreasing, and forces the decreasing current across the air gap at the break in the form of a spark. The

spark at break shows most prominently in circuits having large inductance. For example, the field circuit of a dynamo will produce sparks a centimeter or more in length when broken. The sparking at a dynamo commutator is essentially the *spark at break*.

While the spark lasts, a considerable current will continue to flow in the interrupted circuit on account of the comparatively

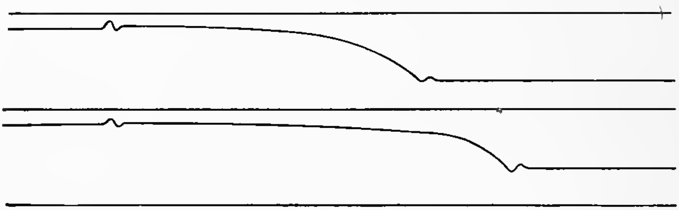

Fig. 324.

low resistance of the heated air gap. The duration of this current and the law of its decadence, which varies with the metal of which the terminals are made and with other conditions, may be studied by means of a galvanometer of the type described in Art. 491. Figure 324 is the reproduction of the photographic record of such an instrument. It represents the dying away of current after breaking circuit (*a*) with copper terminals, (*b*) with brass terminals.

554. Lightning arresters. — The protective action of a lightning arrester depends upon self-induction. The line wires coming in to an instrument or dynamo L (Fig. 325) are connected to two insulated metal blocks *a*, *c*, which are very near to a metal block *b*, connected to earth. The adjacent edges of the blocks *a*, *b*, *c* are serrated to facilitate the formation of sparks between them. At the time of the lightning stroke, the current coming in on the lines

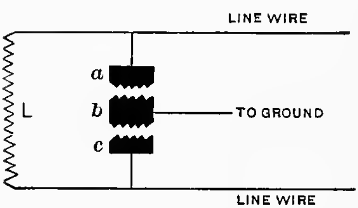

Fig. 325.

increases very rapidly, and the e. m. f. between the blocks *a*, *c* reaches such a large value, in causing the rapid increase of current through the inductive circuit L, that the air gaps are broken down, electrically speaking, and the greater part of the lightning stroke is carried across from *a* to *c*, or from *a* and *c* to *b*, and thence to ground. In case the normal e. m. f. between *a* and *c*, due to the dynamo, is sufficient to maintain the spark, or *arc* as it is called when thus maintained, across the air gaps, an automatic device actuated by an electromagnet is arranged to move the blocks farther apart after the lightning stroke. Another means for stopping the spark after the lightning stroke is to provide an electromagnet which forms an intense magnetic field in the spark gap. This field is arranged to be at right angles to the spark. The spark, being a *current element*, is pushed sidewise by the field (see Art. 353) and thus literally blown out. Such an arrangement is called a *magnetic blow-out*.

555. Non-inductive coils. — For many purposes it is necessary to have coils of which the inductance is negligibly small. A circuit arranged as shown in Fig. 326 has very small inductance. This is evident when we consider that the two oppositely directed currents, side by side, produce only a weak magnetic field in the surrounding region, so that but little kinetic energy is associated with even a strong current. Therefore, from the equation $W = \frac{1}{2} Li^2$, L must be small. The circuit shown in Fig. 326 may be wound on a spool without essentially altering the conditions. A coil of wire wound in this way is called a *non-inductive coil* or an *inductionless coil*. The wire which is used for the various resistances in a resistance box is, in the better class of apparatus, wound in non-inductive coils.

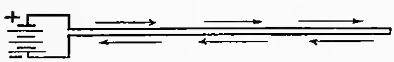

Fig. 326.

556. The induction coil. — This apparatus depends in its action upon the mutual induction of two coils. It consists of a primary coil, usually of coarse wire, wound upon a bundle of soft iron wires AB (Fig. 327); and a secondary coil ss of a large number of turns of fine wire wound over the primary. Figure 328 shows the arrangement of the essential parts. A current being started in the primary, a very large magnetic flux is produced through the iron core. This flux as it increases induces an e. m. f. in the secondary coil equal to $n_{11}\frac{dN}{dt}$, where n_{11} is the number of turns of wire in the secondary coil, and $\frac{dN}{dt}$ is the rate of increase or decrease of the magnetic flux in the core. The primary circuit being rapidly *made* and *broken* at b, the flux through the iron core fluctuates accordingly, producing a fluctuating e. m. f. in the secondary

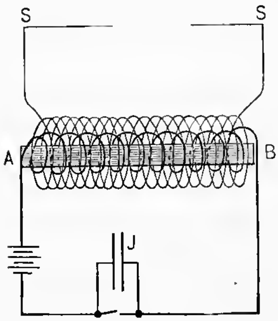

Fig. 327.

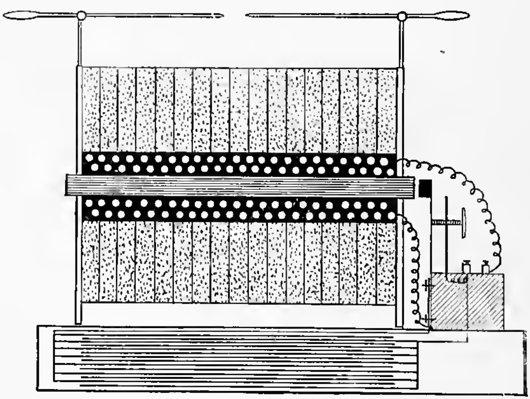

Fig. 328.

coil. A difficulty encountered in the use of the simple *make* and *break* at b is that the iron does not lose much of its magnetism at break, and what it does lose it loses slowly. To

overcome this difficulty the plates of a condenser *J* are connected between the terminals of the break, as shown in Fig. 327. When the circuit is now broken at *b*, the momentum of the current (self-induction) is such as to keep it flowing for a short interval, just as in case of the spark at break, and this persisting current flows into the condenser and charges it. After the momentum of the current is expended in charging the condenser, the condenser discharges itself through the circuit, causing a reversed current, which demagnetizes the iron core of the induction coil with great rapidity, and thus induces an enormous e. m. f. in the secondary coil. When the condenser is connected as shown, the *spark at break* is very much less at *b* than when the condenser is disconnected. The momentum of the current goes to charge the condenser instead of producing the spark at break.

557. The transformer is essentially an induction coil. It consists of two coils, a primary and a secondary, wound upon an iron core. An alternating current sent through the primary magnetizes the core first in one sense and then in the other, and the varying magnetic flux thus produced through the core induces an alternating e. m. f. in the secondary coil.

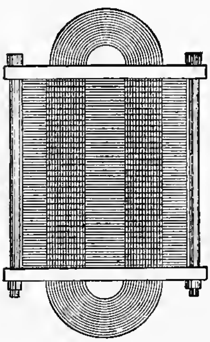

Fig. 329.

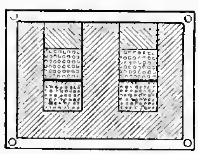

Fig. 330.

The transformers used in commercial work are commonly so constructed that the magnetic circuit of the two coils lies entirely within laminated strips of iron. This is effected by an arrangement of the parts similar to that shown in Figs. 329 and 330.

CHAPTER XIII.

THERMOELECTRIC CURRENTS.

558. Seebeck's discovery. — In 1821 Seebeck found that an electric current is produced in a circuit of two metals when the temperature of one of the junctions of the two metals is different from that of the other junction.

Thus, if the ends of an iron wire are connected with copper wires leading to a galvanometer, the galvanometer indicates a current when one of the junctions is heated. Such an arrangement is called a *thermoelement*.

A highly sensitive galvanometer will indicate currents due to so small a temperature difference as one-thousandth of a degree centigrade between the junctions of a thermoelement. If a number of bars of two metals, AAA..., BBB... (Fig. 331), are connected in series as shown, a much smaller temperature difference between the junctions 1, 3, 5, and 7 on the one hand, and 2, 4, and 6 on the other hand, may be detected. Such an arrangement is called a *thermopile*. Melloni, in his classic experiments on radiant heat, used a thermopile built up with bars of antimony and of bismuth.

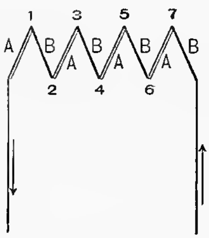

Fig. 331.

559. Peltier's effect. — In 1834 Peltier discovered that heat (aside from that generated in accordance with Joule's Law, Art. 367) is generated or absorbed at a junction of two metals when a current flows from one to the other; generated when the current flows in one direction, and absorbed (that is, the junction becomes cool) when the direction of the current is reversed. For strong currents, this *Peltier effect* is masked by the generation of heat due to electrical resistance, for the rate of genera-

tion of heat by the Peltier effect is proportional to the current, while the rate of generation of heat due to resistance is proportional to the square of the current. If a current from a battery is sent through a thermopile, the two sets of junctions are respectively heated and cooled, in accordance with the Peltier effect, and the thermopile will be found to give a reverse current when disconnected from the battery and connected with a galvanometer.

560. Thomson effect. — Lord Kelvin, from the principles of thermodynamics, was led to suspect the existence of a cooling action (or heating action, according to the direction of current) when an electric current flows along a wire of which the temperature is not uniform. This he found to be the case. In some metals, *e.g.* in copper, the electric current causes an absorption of heat (a cooling effect) at a given point when the current flows in the direction in which temperature is increasing, and *vice versa*. In iron, on the other hand, absorption of heat takes place when the current flows from hot to cold.

561. Thermoelectric power. — Consider a thermoelement of given metals, with one junction at temperature T, the other junction at temperature $T + \Delta T$. Let Δe be the e. m. f. of the element. Experiment shows that the ratio $\frac{\Delta e}{\Delta T}$ approaches a definite limit Q, as ΔT decreases. That is,

$$\frac{de}{dT} = Q. \tag{315}$$

This quantity Q is called the *thermoelectric power* of the two metals at the given temperature T.

The e. m. f. e of a thermoelement, the junctions of which are at temperatures T and T', is $e = \Sigma Q \Delta T$, or

$$e = \int_T^{T'} Q \cdot dT.* \tag{316}$$

* This equation is not a direct result of (315), but depends, in addition, upon the fact that the e. m. f. of a thermoelement, whose junctions are at T and T'', is the sum of the e. m. f.'s of the element when its junctions are at temperatures T and T' and at T' and T'' respectively, T' being a temperature between T and T''.

in which Q is the thermoelectric power (varying with temperature) of the two metals.

562. Thermoelectric diagram. — The ordinates between the various curves (straight lines) in Fig. 332 represent the values

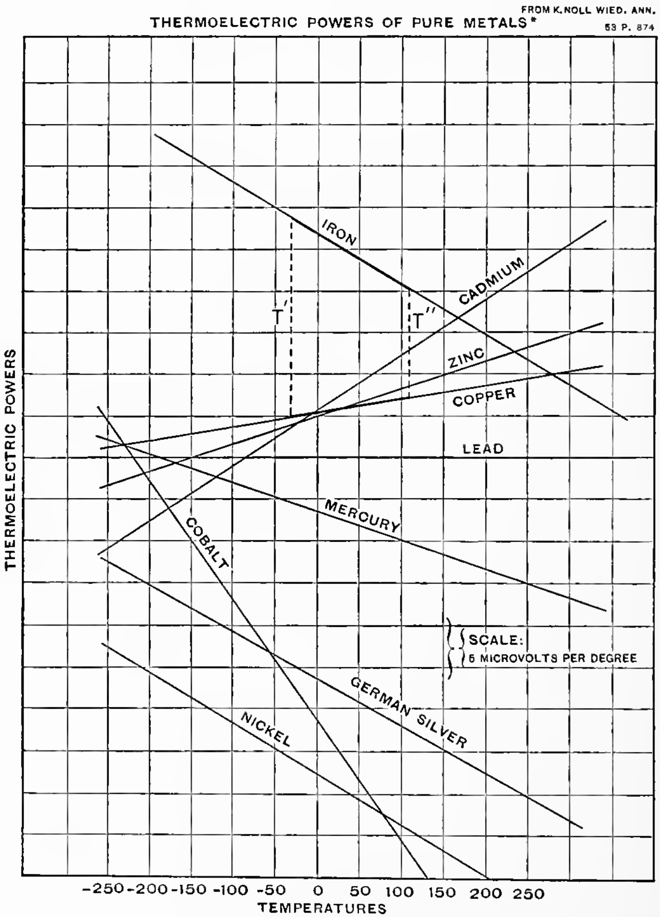

Fig. 332.

of Q in volts per degree for the respective pairs of metals. These lines are called the thermoelectric lines of the respective

metals, and the figure is called the thermoelectric diagram. In the diagram let Q' be the thermoelectric power of iron-lead, and Q'' that of lead-copper, at a given temperature. The thermoelectric power of iron-copper is $Q' + Q''$.

From equation (316) it follows that the e. m. f. of a thermoelement, say of iron and copper, when its junctions are at T' and T'', is represented by the area inclosed between the thermoelectric lines for iron and for copper, and the ordinates whose abscissæ represent the temperatures T_1 and T_2, respectively. This area is heavily outlined in Fig. 332. Areas on opposite sides of the point of intersection of two thermoelectric lines are to be considered as opposite in sign.

563. Neutral temperature of two metals. — The temperatures at which the thermoelectric power of two metals is zero is called their *neutral temperature;* it is the temperature at which their thermoelectric lines intersect.

With one junction of a thermoelement kept at constant temperature, and the other at temperature T, the e. m. f. e of the element is

$$e = a + bT + cT^2, \tag{317}$$

in which a, b, and e are constants depending upon the metals employed, and upon the constant temperature of the one junction.

Proof. — Let n be the neutral temperature, and T_1 the constant temperature of the one junction. The thermoelectric lines being sensibly straight, the inclosed area (Fig. 332) between temperatures n and T_1 is $K(n - T_1)^2$, and the inclosed area between n and T is $K(n - T)^2$, in which K is a constant depending upon the divergence of the thermoelectric lines of the metals. The area between temperatures T_1 and T, which represents the required e. m. f., is the difference of the areas $K(n - T_1)^2$ and $K(n - T)^2$. Upon reduction this expression, $K(n - T_1)^2 - K(n - T)^2$, is found to contain a constant term, viz. $K(n - T_1^2) - Kn^2 (= a)$, a term involving T, and a term involving T^2.

564. Use of thermoelement in the measurement of temperature. — The constants a, b, and c of equation (317) being determined for a given temperature, say 0° C., of one junction of the thermoelement, the temperature T of the other junction may be calculated when e has been observed. The thermoelement furnishes in this way a very convenient means for the determination of temperature, a method which is peculiarly adapted to very high temperatures and to very low temperatures. The derivation of equation (317) is based upon the assumed straightness of the thermoelectric lines for the various metals; this is strictly not the case, so that equation (317) is not rigorously exact.

By the selection of a thermoelement, for which the assumption is approximately justified for the range of temperatures to be measured, and by calibrating the element by determining the e. m. f. which is produced when one junction is maintained at the temperature of melting ice, while the other is brought to some other known temperature, such as that of melting silver for very high temperatures, or of melting mercury, or of boiling oxygen for very low temperatures, trustworthy results have been obtained by this method. For the measurement of very high temperatures the element chosen generally consists of pure platinum and an alloy of platinum-iridium or platinum-rhodium.

565. Thermoelectric batteries as a substitute for voltaic cells. — Although the e. m. f. of a single thermoelectric element is small, it is possible by connecting many in series to construct a battery with an e. m. f. of several volts. The metals most suitable for this purpose, on account of their ability to stand a higher temperature than the more sensitive combination of antimony-bismuth, is German silver and an alloy of antimony and zinc. The e. m. f. of a single element of this kind when one junction is at ordinary temperatures and the other is heated as hot as can safely be done, is about 0.04 volt. To facilitate the simultaneous heating of the alternate junctions, the

elements are connected in the form of rings (Fig. 333), and these rings are piled one upon another so as to form a hollow cylinder. The inner face of this cylinder, which is made up of

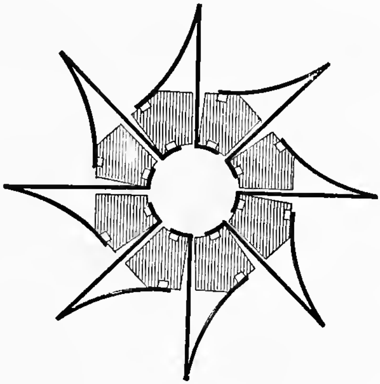

Fig. 333.

the junctions to be heated, is subjected to the action of a gas flame. This construction is due to Clamond.

In this method of producing current, the source of energy is the flame. The heat losses, however, are of necessity so great as to more than make up for the cheapness of the reagents employed in the thermobattery.

For a fuller discussion of thermoelectric currents see J. J. Thomson, *Elements of Electricity and Magnetism*, pp. 493–505. The student also is referred to more extended treatises for descriptions of the more obscure electric and magnetic phenomena, such as piezoelectricity, pyroelectricity, Hall's phenomenon, Kerr's effect, Kundt's effect, etc., etc. Some of these phenomena indicate the failure in ponderable matter of the principle of superposition as stated in Arts. 343 and 450.

CHAPTER XIV.

SOME PRACTICAL APPLICATIONS OF ELECTRICITY AND MAGNETISM.

Electric Signaling.

566. Simple telegraphing. — The electromagnet is much used for signaling between distant stations. An insulated wire leads from one station to the other and back. The ground is generally used instead of a return wire. An electric current from a battery or other source is sent intermittently through this circuit by operating, at one station, a *key* which makes and breaks the circuit. This current excites an electromagnet at the other station, and the armature of this electromagnet either makes a graphical record or produces sound signals. Messages are sent by making use of a code of signals.

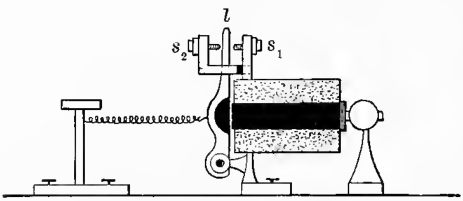

Fig. 334.

Relays; sounders. — A *relay* consists of an electromagnet, usually wound with fine wire, which actuates a light lever l (Fig. 334), and this lever is arranged to open and close a distinct electric circuit as it moves to and fro between the stops s_1, s_2. A *sounder* is an electromagnet, usually wound with coarse wire, which actuates a rather heavy lever l (Fig. 335), arranged to produce audible clicks as it moves to and fro between stops. On long telegraph lines a relay

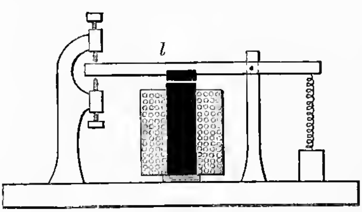

Fig. 335.

and a key at each station are in circuit with the line. When a key at any station is operated, all the relays act simultaneously. At each station a sounder is actuated by one or two cells of battery under the control of the make and break device of the relay. (See Fig. 336.) The battery which furnishes the line current may be located at any convenient place along the line.

The *Morse recorder* consists of an electromagnet which actuates a lever carrying a pencil or stylus under which a strip of paper is moved along by clockwork. When the electromagnet is excited, the point is brought against the moving paper, making a mark or indentation. When the exciting current ceases, the lever is pulled back by a spring lifting the pencil from the paper. In addition to the ordinary keys, sounders, relays, and recorders, used in telegraphy, a number of ingenious devices are employed in the modern service. Some of these are described in the following articles.

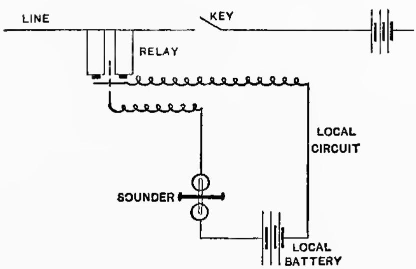

Fig. 336.

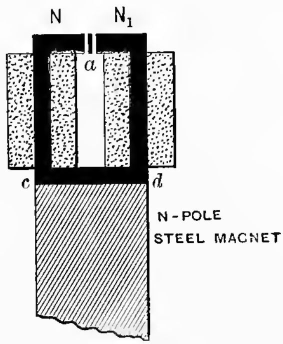

Fig. 337.

567. The polarized relay. — An electromagnet NN_1cd is mounted, as shown in Fig. 337, upon one pole of a strong U-shaped magnet of steel. The cores are of soft iron. A light bar of soft iron a is pivoted at p, Fig. 338, extends through a slot in the south pole SS of the steel magnet, and on between the poles NN_1 of the electromagnet, and plays between the stops p' and p''. When no current flows through the coils of the

electromagnet, NN_1 are north poles of equal strength. The bar a is magnetized inasmuch as it bridges between the poles SS and NN_1 of the steel magnet. The portion of a between NN_1 is of south polarity and is attracted by both N and N_1, but being nearer to N, as shown, it is held against the stop p''. When a current is sent in one direction through the coils of the

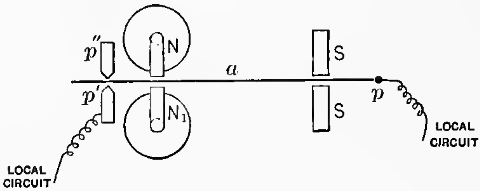

Fig. 338.

electromagnet, the pole N_1 is strengthened, the pole N is weakened, and the bar a is pulled against the stop p', closing the local circuit. When a current is sent in the other direction through the coils of the electromagnet, N is strengthened and N_1 is weakened, and the bar a remains against the stop p''. Such an instrument is called a *polarized relay.*

568. Diplex telegraphy. — The sending of two messages in the same direction over one line wire simultaneously is known as *diplex telegraphy.* This is accomplished as follows: An ordinary relay is provided with a strong spring so that its lever responds only to a *strong* current (in either direction, of course). This relay and a polarized relay are put in circuit at the receiving station. At the sending station are two keys. One of these keys is arranged to vary the *strength* of current in the line (never actually breaking circuit) by throwing a number of batteries in and out of circuit as it is operated. The ordinary relay responds to the movements of this key. The other key is arranged to reverse the *direction* of the line current as it is operated, the line current being in one direction while this key is down and in the other direction while it is up. The polarized relay responds to the movements of this key.

569. Duplex telegraphy. — The sending of two messages in opposite directions over one line simultaneously is known as *duplex telegraphy.* This is accomplished as follows:

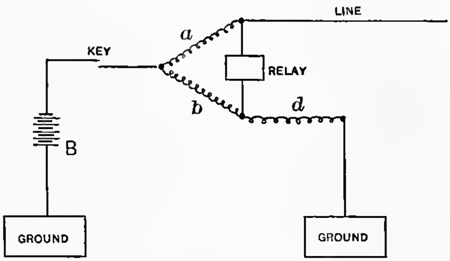

Fig. 339.

Figure 339 represents the arrangement of apparatus at one station. A similar arrangement is installed at the other station. Let c be the total resistance of the line through the distant station to the ground. The resistances a, b, c, d form a Wheatstone's bridge. When these resistances are so adjusted that $ad = bc$, the key may be pressed without sending current through the relay. When the key is pressed, however, current flows over the line to the other station, and it is easily seen from the figure that a line current coming to a station divides and flows in part through the relay at that station. Therefore the relay at each station responds to the movements of the key at the other station.

570. Quadruplex telegraphy. — The sending of two messages each way over one line simultaneously is known as *quadruplex telegraphy.* This is accomplished by combining the arrangements for diplex and duplex telegraphy The single key represented in Fig. 339 is replaced by two keys, one for reversing the current, and the other for altering its strength; and the single relay is replaced by two, one an ordinary relay with a stout spring, and the other a polarized relay. The polarized relay at each station responds to the reversing key at the other station,

and the common relay at each station responds to the key at the other station which alters the strength of the current.

571. Submarine cables for telegraphy are made by covering the conductor with an insulating sheath, the whole being protected by an outer covering of hemp and iron wire (see Fig. 340).

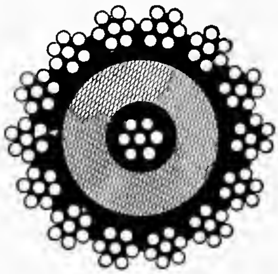

Fig. 340.

The conductor itself is made of several strands of copper twisted together. Thus low resistance and great flexibility are secured.

The conductor and the metal sheath of a cable, together with intervening insulator, constitute a condenser of large electrostatic capacity.

When a battery is connected so as to send a current through a cable we have very different conditions at the two ends. At

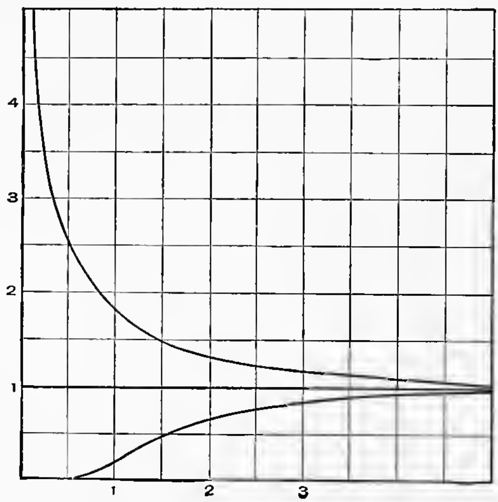

Fig. 341.

the end near the battery the current is at first very large, while at the far end there is at first no appreciable current whatever.

As the cable becomes charged, the former current diminishes while the latter rises, both reaching a common final value.

Figure 341 shows the current at the near and at the far end as a function of the time.

At intermediate points within the cable the current has intermediate values, so that we have, for a given instant, a distribution of current values which can be expressed by means of a curve. The form of this curve changes with the time as shown in Fig. 342.* This figure, in which ordinates are intensities of current

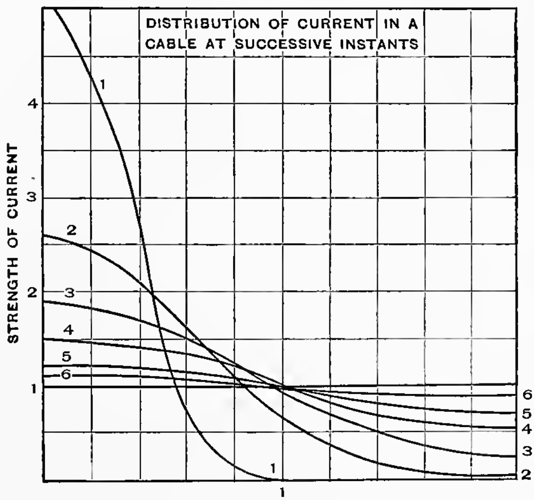

Fig. 342.

and abscissæ are distances from the near end of the cable, gives the curve of distribution at successive moments of time, also its final form, which is that of a horizontal line.

The excess of current at the near end goes to furnish the static charge which is distributed along the conducting wire of the cable as the potential of this wire rises. After disconnecting the battery from the near end of the cable, breaking circuit,

* Figures 341 and 342 are from data given by Froehlich, Handbuch der Elektricität und des Magnetismus, pp. 365 *et seq.*

the current at the far end drops slowly to zero as the conducting wire loses its static charge.

This property of cables produces modifications in the character of the momentary currents used in signaling, so that the

Fig. 343.

ordinary method of relay and sounder is no longer applicable. If, for example, a telegraph circuit be closed for a fraction of a second by means of the operating key, to produce what is called a dash, the effect produced, both at the sending and the receiving station of a line of negligible electrostatic capacity and induction, could be represented by means of the diagram in Fig. 343, in which the ordinates are current strength, and abscissæ are times. At the far end of a cable, however, we might expect, from Figs. 341 and 342, that the signal would be modified. The series of tracings shown in Fig. 344 illustrate this point. They were made in the establishment of Siemens and Halske in Berlin. The line *a* shows the form of signal at the near end

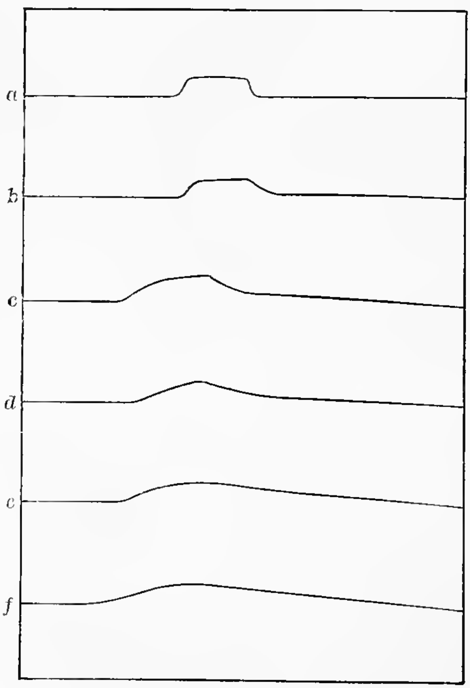

Fig. 344.

of an artificial cable; *f*, its form at the far end. The intervening tracings refer to intervening points.

Experiments made upon the underground cables of the German telegraph system have furnished, in fact, results which agree with the above. Figure 345 shows the form of an ordinary make-and-break signal sent through 751 kilometers of such cable. It is reproduced from the trace upon smoked paper

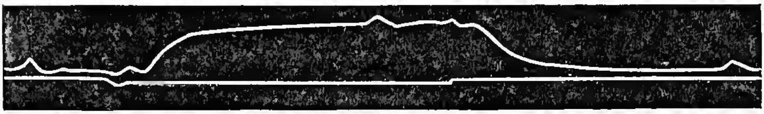

Fig. 345.

of a recording instrument. The effect of self-induction is to decrease very slightly this tendency to sluggish rise and fall of current.

Overhead lines act in a similar manner; but unless the line is very long, the current is prompt enough in its fluctuations for telegraphy with Morse instruments at the rate of two hundred or more signals per minute.

In the early days of ocean telegraphy the signals were received by means of a mirror galvanometer. When the battery at the far station is connected for a very short time, the current at the near station rises to a small value only, and produces a small deflection on the galvanometer. When the battery is connected for a longer time, the current at the near station rises to a larger value, and produces a greater deflection. The *dots* and *dashes* of the Morse code are thus distinguished, the deflections of the galvanometer being indicated, using lamp and scale, by the movement of a spot of light.

572. The siphon recorder, which has largely supplanted the ordinary galvanometer in submarine signaling, is essentially a D'Arsonval galvanometer (see Art. 366), to the suspended coil of which is fixed a fine glass tube in the form of a siphon,

arranged to discharge ink upon a strip of paper moved along by clockwork (Fig. 346). The end of this siphon moves sidewise as the coil is deflected by the cable current, and a wavy line upon the paper is the result. The dots and dashes are distinguished by the amplitude of the corresponding waves in this ink line.

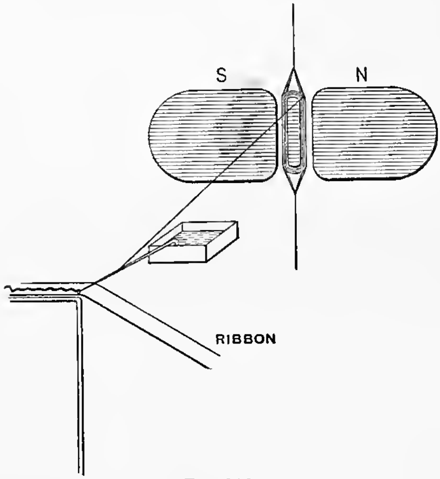

Fig. 346.

Owing to the sluggish rise and fall of current in an ocean cable, the duplex arrangement described in Art. 569 has to be modified for cable work. The resistance *d* (Fig. 339) is replaced by an *artificial cable*, an arrangement simulating exactly the electric behavior of the cable, so that the current sent out from a station rises and falls similarly in *c* and *d* (Fig. 339), leaving the receiving instrument at the sending station unaffected except it be by currents coming from the other end of the cable.

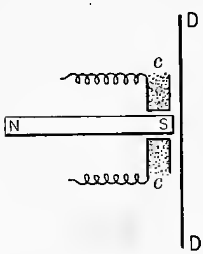

Fig. 347.

573. The telephone consists of a thin sheet-iron diaphragm *DD* (Fig. 347) very near to which is one end of a steel magnet *NS* with a coil of fine insulated wire *cc*.

Action of telephone as transmitter. The coil *cc* being near the end of *NS*, only a portion of the magnetic flux through the magnet passes through the coil. If *DD* is moved nearer to *S*, a greater portion of the magnetic flux through the magnet will pass through *cc*, and *vice versa*; but since any change of magnetic flux through *cc* induces an e. m. f. in *cc* and this in

turn produces current in any circuit with which *cc* is connected, therefore *any to-and-fro motion of DD produces corresponding currents in the coil cc.*

Action of telephone as receiver. If a current passes through *cc* first in one direction and then in the other, the magnet *NS* will be correspondingly weakened and strengthened, and the force with which the magnet attracts *DD* will vary accordingly, *causing DD to move to and fro in unison with the currents flowing in cc.*

Consider two telephones connected in circuit. A sound striking the diaphragm of one will cause it to vibrate. This telephone acting as a *transmitter* produces currents which cause a similar vibratory motion of the diaphragm of the other telephone, acting as a *receiver;* and this motion of the diaphragm of the receiver telephone reproduces the original sound.

The telephone is an instrument of remarkable delicacy. The amount of current necessary to produce an audible vibration of the diaphragm of a receiving telephone depends upon the frequency. Ferraris, who has made an investigation of this subject, found for the minimum current necessary to produce a sound the following values: *

TABLE.

Pitch in Single Vibrations per Second.	Current in Amperes.
269	2.3×10^{-10}
352	1.7×10^{-10}
440	1.0×10^{-10}
523	0.7×10^{-10}
594	0.5×10^{-10}

These results are embodied in the curve in Fig. 348.

* Winkelmann, Handbuch der Physik, Vol. III. (2), p. 525.

The distance through which the diaphragm moves to produce an audible sound has been estimated at a ten-millionth of one centimeter.*

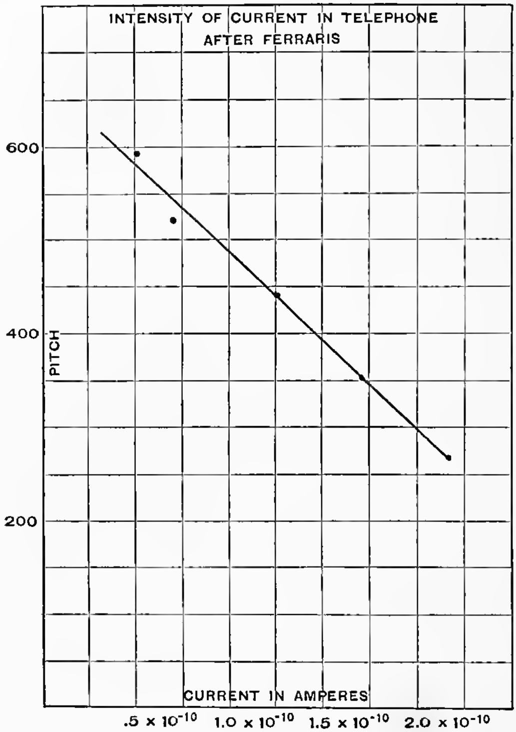

Fig. 348.

574. Preece's law. — Telephonic transmission is limited as to distance by the modification of the current waves within the wire, which always consist of a more or less complex set of sine waves superimposed. The cause which brings about these modifications is identical with that which interferes with telegraphy through cables, *i.e.* the capacity of the circuit. Preece † has shown that the distance to which intelligible speech can be

* Franke, Electrotechnische Zeitschrift, Vol. XI., p. 289.

† Preece and Stubbs, Manual of Telephone, p. 472.

transmitted, as had been previously shown by Kelvin to be the case in telegraphy, is *approximately inversely proportional to the product* $J \times R$, where J is the capacity of the line, and R is its resistance.

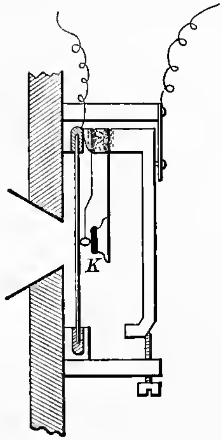

Fig. 349.

575. The carbon transmitter. — The efficiency of telephonic transmission is very low. It has been estimated by Vierordt, for example, that the intensity of the sound which issues from the receiver possesses less than $\frac{1}{500}$ of the intensity of that which falls upon the diaphragm of the transmitter. The currents produced by a telephone acting as a transmitter, as we have already seen, are very weak at best. The carbon transmitter is an arrangement to improve the conditions of telephonic service by the production of comparatively strong currents. Its action is as follows:

The current from a strong battery passes through the primary of a small induction coil and through a mass of powdered carbon K (Fig. 349), which is packed into the form of a button or disk, behind a vibrating diaphragm. The electrical resistance of the carbon varies with the pressure exerted upon it by the moving diaphragm, and the battery current fluctuates correspondingly. This fluctuating battery current induces e. m. f.'s in the secondary of the induction coil, which produces the desired current in the wire leading to the telephone receivers. Figure 350 shows the arrangement of the circuits.

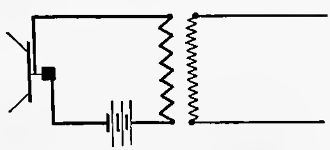

Fig. 350.

576. Dynamos and motors. — The essential features of the direct current dynamo and of the alternating current dynamo are described in Arts. 534 and 535. Such machines enable

mechanical energy to be utilized directly for the maintenance of electric currents. A dynamo is a reversible machine. That is, it may be used as a motor if a current from some outside source is forced through it in a direction opposite to that in which it tends to produce current. Direct current dynamos and motors are identically similar. Some alternating current motors are, however, very different from the alternating current generator. To consider the very numerous forms of dynamo and motor which have been devised to fulfil the demands of modern engineering practice lies quite beyond the scope of the present volume.

577. Electric lighting. — An electric lamp is a device for illumination by means of the incandescence of a high resistance portion of an electric circuit, made of very refractory material. This, being heated to a high temperature by the current, gives off light. The electric lamps in practical use may be classified either as incandescent (or glow) or as arc lamps.

578. The glow lamp consists of a fine filament of charred material upon which a further deposit of dense carbon is formed

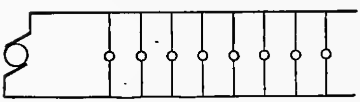

Fig. 351.

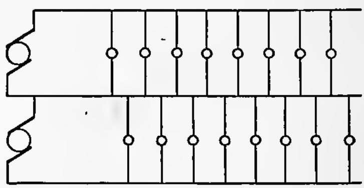

Fig. 352.

by heating it in the vapor of gasolene. The heating is accomplished by sending a current through it. This filament is then mounted in a glass bulb with lead-in wires connecting with the ends of the filament. The air is exhausted from the bulb by means of a mercurial air pump or of a mechanical pump of special construction. It has been found possible to construct such pumps capable of reducing the pressure to about $\frac{1}{25000}$ of an atmosphere.

Incandescent lamps are ordinarily connected in multiple as shown in Fig. 351. The dynamo maintains a constant e. m. f. between the mains, and each lamp, independently of the others, takes an amount of current equal to the quotient of this e. m. f. divided by the resistance of the lamp.

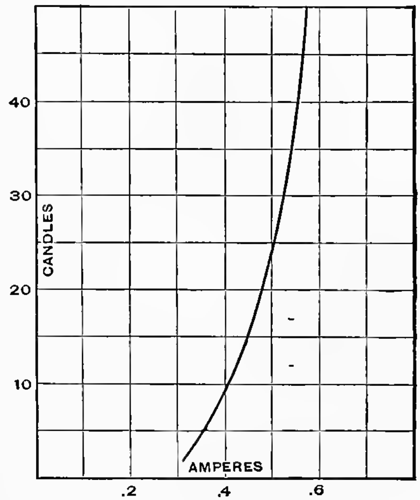

Fig. 353.

Frequently the arrangement indicated in Fig. 352, which is called the *three-wire* system, is employed for the purpose of saving copper. The amount of light obtained from an incandescent lamp increases more rapidly than the current supplied to the filament, as will be seen from Fig. 353. It increases more rapidly also than the difference of potential at the terminals of the lamp, as shown in Fig. 354.

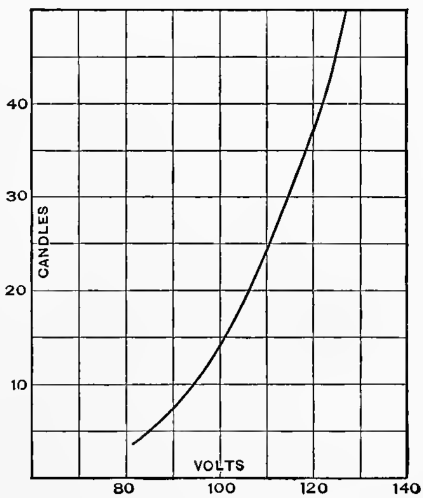

Fig. 354.

The candle power of incandescent lamps, per watt of power expended in them, varies with the temperature of the filament. The precise nature of this variation cannot be stated, because of the difficulty of measuring such high temperatures. The relation of candle power per watt to the candle power of the lamp can, however, be

derived from measurements of candle power, current, and voltage. The result of such a measurement is given in Fig. 355.*

The energy value of the light given by an incandescent lamp is very small at best. Under the conditions of normal use it is only three or four per cent of the power expended in maintaining the incandescence. This ratio, which is called the radiant efficiency of the lamp, is also a function of the temperature of the filament, and therefore of the candle power. The increase of efficiency with candle power is shown in Fig. 356.

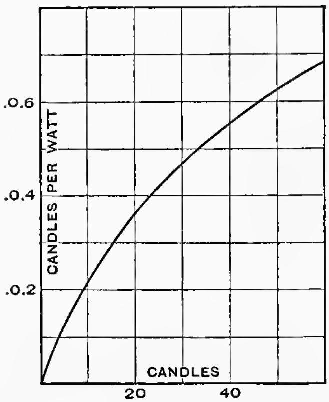

Fig. 355.

579. **The arc lamp** is a mechanism for automatically moving two carbon rods so that an electric arc† between them may be kept steady. The action of the mechanism commonly used is as follows:

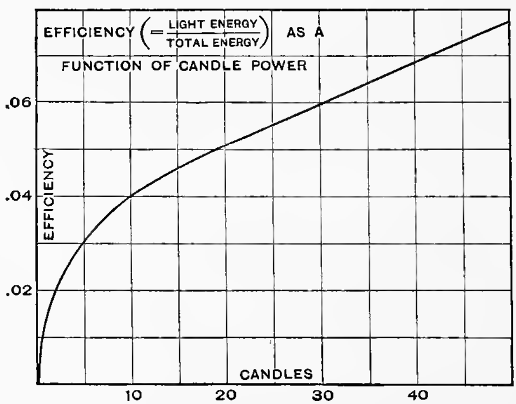

Fig. 356.

* See further, Nichols, Laboratory Manual, Vol. II., p. 322 *et seq.*
† See Art. 499.

The current comes into the lamp and divides as shown in Fig. 357. A very small portion of the current flows through a shunt coil *B* without passing through the arc, and the remainder flows through the coil *A*, thence through the arc. An iron rod *AB* passing loosely into the coils *A* and *B* is carried upon one end of a lever which turns about the point *p*. The other end of this lever is provided with a clutch *c*, through which a smooth brass rod *bb* passes. This clutch *c* is so constructed that it loses its hold on the rod *bb* when the iron rod *AB* is raised; thus allowing the carbons to move closer together.

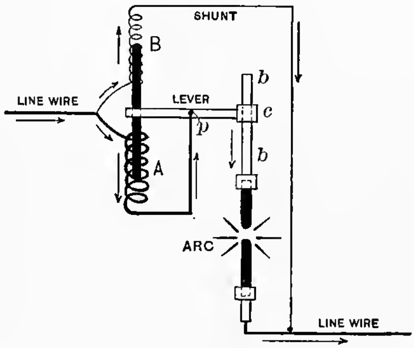

Fig. 357.

Each of the coils *A* and *B* acts to pull the rod *AB* into itself. A spring attached to the lever is so adjusted that when the arc is burning properly the combined action of this spring and the two coils *A* and *B* is to hold the lever in such a position that the clutch grips the brass rod *bb*. As the arc continues to burn, the carbons are slowly consumed, causing the gap between the carbon tips to widen. This increases the resistance of the arc and causes a greater portion of the current to flow through the shunt coil *B*, which pulls up the iron rod, moves the lever, releases the clutch, and allows the rod *bb* to fall slightly, bringing the carbons again to the proper position.

Arc lamps are usually connected in series, and the dynamo is arranged to maintain a constant current through the circuit. A multiple arrangement is, however, sometimes employed.

Arc lamps give from one to three candles of light per watt of power expended in them. Their radiant efficiency, *i.e.* the ratio of the energy value of the light to the total energy, is correspondingly higher than that of the incandescent lamp. The

efficiency depends upon the diameter of the carbons. Marks,* for example, found the following values:

Diam. of Carbons.	Radiant Efficiency (Mean).
.625 cm.	16.60 per cent.
1.250 cm.	12.66 per cent.
2.098 cm.	6.87 per cent.

The length of the arc, other conditions remaining constant, increases with the current. Upon increasing the current from 6 amperes to 13 amperes, Marks found the arc to change (with constant e. m. f.) from 0.217 cm. to 0.417 cm. An increase in the potential (with constant current) from 40 volts to 60 volts changed the length of arc from 0.182 cm. to 0.540 cm.

580. The electric furnace. — The working of very refractory metals and the forcing of certain chemical actions require extremely high temperatures, such as can be produced only in the electric furnace. This furnace consists of a chamber filled loosely with powdered coke, through which a strong electric current is sent. The heat generated by the current soon brings the whole mass to an extremely high temperature. The current is always introduced by means of carbon rods. Carbon, indeed, is the only conducting material which is sufficiently refractory for this purpose.

In the manufacture of *calcium carbide*, the electric furnace is charged with a mixture of powdered coke and lime. In the manufacture of silicon carbide (*carborundum*), the furnace is charged with a mixture of powdered coke and sand.

For very many operations an inclosed arc forms the most convenient and effective electric furnace. The furnace of Moissan (Fig. 358) is of this type. It consists of a graphite crucible

* Marks, Trans. of the American Institute of Electrical Engineers, Vol. 7, p. 175.

contained within massive walls of lime or fire-brick. The current is introduced by means of carbon rods, through apertures in the ends of the furnace, as shown in the illustration.

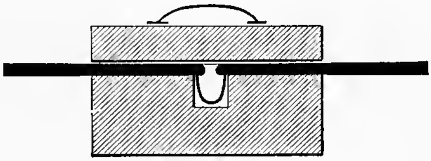

Fig. 358.

Other applications of electricity to the arts, many of which, such as electro-metallurgy, electrotyping, electric welding, the electric transmission of power, etc., are of great practical importance, cannot be considered within the limits of the present work.

CHAPTER XV.

SYSTEMS OF ELECTRIC AND MAGNETIC UNITS; THE ELECTROMAGNETIC THEORY OF LIGHT.

581. Résumé of electric and magnetic equations.* — In order to obtain a general view of the relations of the various electric and magnetic quantities a number of equations are here collected, and a brief description is given of each. For the sake of generality the factors μ and k (see Arts. 446, 520) are retained throughout.

It is important to distinguish those equations which are *independent*, those which are mere *definitions*, and those which are *derived*. Those equations only are independent which formulate independent experimental facts.

582. Independent equations.

$$F = \frac{1}{k} \frac{Q_1 Q_{11}}{d^2}. \qquad (1)$$

In which F is the force with which two electric charges Q_1 and Q_{11} repel at distance d from each other in a medium of specific inductive capacity k. This equation formulates Coulomb's Law. (See Arts. 445, 446.)

$$F = \frac{1}{\mu} \frac{m_1 m_{11}}{d^2}. \qquad (2)$$

In which F is the force with which two magnetic poles m_1 and m_{11} repel at distance d from each other in a medium of which the magnetic permeability is μ. This equation formulates Cavendish's Law. (See Arts. 330, 331.)

$$F = Qe.\dagger \qquad (3)$$

In which F is the force which acts upon a charge Q when placed at a

$$F = mf. \qquad (4)$$

In which F is the force which acts upon a magnetic pole of strength m

* For brevity the equations in this chapter are numbered consecutively, irrespective of the numbers previously assigned to them.

† Heretofore the letter f has been used for intensity of electric field and for intensity of magnetic field alike, and N has been used for both electric flux and magnetic flux.

point in an electric field of intensity e. Equation (1) may be written

$$F = Q_1\left(\frac{1}{k}\frac{Q_{11}}{d^2}\right),$$

so that putting

$$e \equiv \frac{1}{k}\frac{Q_{11}}{d^2},$$

we have equation (3). However, there are other electrical fields than those which emanate from electric charges

$$\left(= \frac{1}{k}\frac{Q}{d^2}\right),$$

and equation (3) has greater generality than is compatible with its derivation from equation (1), using the definition

$$e \equiv \frac{1}{k}\frac{Q}{d^2}.$$

(See Arts. 447, 449.)

when placed at a point in a magnetic field of intensity f. Equation (2) may be written

$$F = m_1\left(\frac{1}{\mu}\frac{m_{11}}{d^2}\right),$$

so that putting

$$f \equiv \frac{1}{\mu}\frac{m_{11}}{d^2},$$

we have equation (4). However, there are other magnetic fields than those which emanate from magnetic poles

$$\left(= \frac{1}{\mu}\frac{m}{d^2}\right),$$

and equation (4) has greater generality than is compatible with its derivation from equation (2), using the definition

$$f \equiv \frac{1}{\mu}\frac{m}{d^2}.$$

(See Arts. 335, 336.)

583. Definitions.

$$E \equiv \Sigma e \,.\, \cos\epsilon \,.\, \Delta s.\text{*} \qquad (5)$$

That is, the electromotive force or e. m. f. E along any line is *defined* as the line integral of e along that line. (See Arts. 315, 466.)

$$\Omega \equiv \Sigma f \,.\, \cos\epsilon \,.\, \Delta s.\text{*} \qquad (6)$$

That is, the magnetomotive force, or m. m. f. Ω along any line is *defined* as the line integral of f along that line.† (See Art. 506.)

$$N \equiv \Sigma ke \,.\, \cos\epsilon \,.\, \Delta S. \qquad (7)$$

That is, the electric flux N across a surface is *defined* as the surface integral of ke taken over the surface. (See Arts. 320, 452.)

$$M \equiv \Sigma \mu f \,.\, \cos\epsilon \,.\, \Delta S. \qquad (8)$$

That is, the magnetic flux M across a surface is defined as the surface integral of μf taken over the surface. (See Art. 320.)

* The equation $E \equiv \Sigma e \cdot \cos\epsilon \cdot \Delta s$ reads: E STANDS FOR $\Sigma e \cdot \cos\epsilon \cdot \Delta s$. Such an equation ranks, of course, as an *independent* equation so far as algebra is concerned. There is, however, not the least physical significance to such an equation.

† Many of the following magnetic equations have not been previously discussed. The proof of these magnetic equations is in each case *identical* to the proof of the corresponding electric equation.

584. Derived equations.

$$e = \frac{1}{k}\frac{Q}{d^2}. \qquad (9)$$

In which e is the intensity of the electric field at a point distant d from a concentrated charge Q in a medium of specific induction capacity k. It is derived from equations (1) and (3). (See Art. 449.)

$$f = \frac{1}{\mu}\frac{m}{d^2}. \qquad (10)$$

In which f is the intensity f of the magnetic field at a point distant d from a concentrated pole m in a medium of permeability μ. It is derived from equations (2) and (4). (See Art. 338.)

$$N = 4\pi Q. \qquad (11)$$

In which N is the total outward electric flux from a charge Q. It is derived from equations (9), viz. $ke = \frac{Q}{d^2}$, and (7). (See Art. 462.)

$$M = 4\pi m. \qquad (12)$$

In which M is the total outward magnetic flux from a pole m. It is derived from equation (10), viz. $\mu f = \frac{m}{d^2}$, and (8).

Remark. — Consider an electric tube of flow through which the electric flux is N. The electric charge associated with one end of this tube is $\frac{1}{4\pi}N$ by equation (11). If the tube is endless, then $\frac{1}{4\pi}N$ is the charge which would be associated with one end of it were it cut across. We shall hereafter speak of the charge associated with one end of an electric tube as the *strength* of the tube.

Remark. — Consider a magnetic tube of flow through which the magnetic flux is M. The magnetic pole associated with one end of this tube is $\frac{1}{4\pi}M$ by equation (12). If the tube is endless, then $\frac{1}{4\pi}M$ is the pole strength which would be associated with one end of it were it cut across. We shall hereafter speak of the pole strength associated with one end of a magnetic tube as the *strength* of the tube.

$$E = \frac{1}{k}\frac{Q}{d}. \qquad (13)$$

In which E is the e. m. f. from a point distant d from a charge Q, to infinity in a medium of specific induction capacity k. It is derived from equation (9) using (5). The e. m. f. from a given point to infinity, or to any arbitrarily chosen region, is called the *electric potential* at the point when it is independent of the path over which the e. m. f. is reckoned.

(See Art. 466.)

$$\Omega = \frac{1}{\mu}\frac{m}{d} \qquad (14)$$

In which Ω is the m. m. f. from a point distant d from a pole m to infinity in a medium of permeability μ. It is derived from equation (10), using (6). The m. m. f. from a point to infinity, or to any arbitrarily chosen region, is called the *magnetic potential* at the point when it is independent of the path over which the m. m. f. is reckoned.

$$W = QE\,^*$$ (15)

In which W is the work done in moving a charge Q along a line over which the e. m. f. is E. It is derived from equation (3), using (5). (See Art. 467.)

$$W = m\Omega.^*$$ (16)

In which W is the work done in moving a magnetic pole m along a line over which the m. m. f. is Ω. It is derived from equation (4), using (6).

$$W = \frac{1}{8\pi} NE;$$ (17)

or using equations (11) and (12):

$$W = \frac{1}{8\pi} M\Omega.$$ (18)

$$W = \tfrac{1}{2} QE.$$ (19)

In which W is the total energy of an electric tube through which the electric flux is N, or at the ends of which are charges $\pm Q$, E being the e. m. f. along the tube. Equations (17) and (19) hold for endless electric tubes also. Compare remarks following equation (11). (See Art. 474.)

$$W = \tfrac{1}{2} m\Omega.$$ (20)

In which W is the total energy of a magnetic tube through which the magnetic flux is M, or at the ends of which are poles $\pm m$, Ω being the m. m. f. along the tube. Equations (18) and (20) hold for endless magnetic tubes also. Compare remarks following equation (12).

$$W = \frac{k}{8\pi} e^2.$$ (21)

In which W is the energy *per cubic centimeter* at a point in an electric field, e being the intensity of the field at the point, and k the specific inductive capacity of the dielectric. This equation is derived from (17). See Elements of Electricity and Magnetism, J. J. Thomson, p. 70.

$$W = \frac{\mu}{8\pi} f^2.$$ (22)

In which W is the energy *per cubic centimeter* at a point in a magnetic field, f being the intensity of the field at the point, and μ the permeability of the medium. This equation is derived from (18).

Proposition. — The rate $\frac{dW}{dt}$ at which the energy of an electric tube increases is $\frac{dW}{dt} = E\frac{dQ}{dt}$, in which E is the e. m. f. along the tube, and Q is the charge at one of its ends.

Proof. — Equations (17) and (19) express the total energy of such a tube. Imagine the electric field at every point in such

* The e. m. f. in equation (15) is understood to be due to *other charges* than Q which is being moved; and the m. m. f. in equation (16) is understood to be due to something besides the pole m which is being moved.

a tube to be everywhere increased in intensity in a given ratio. Then N (and also Q), being the surface integral of ke (see equation (7)), and E, being the line integral of e (see equation (5)), will be both increased in that ratio. That is, E/Q remains constant, so that we may write

$$E = bQ, \tag{a}$$

in which b is a constant of which the value does not concern us. Substituting this value of E in (19), we have for the energy of the electric tube:

$$W = \tfrac{1}{2} bQ^2, \tag{b}$$

whence

$$\frac{dW}{dt} = bQ\frac{dQ}{dt}. \tag{c}$$

Substituting E for bQ from (a), we have $\dfrac{dW}{dt} = E\dfrac{dQ}{dt}$.

Q.E.D.

Therefore we have the equations:

$$\frac{dW}{dt} = E\frac{dQ}{dt}. \tag{23}$$

In which $\dfrac{dW}{dt}$ is the rate of increase of the energy of an electric tube of strength Q, and E is the e.m.f. along the tube. The tube may or may not be endless.

$$\frac{dW}{dt} = \Omega\frac{dm}{dt}. \tag{24}$$

In which $\dfrac{dW}{dt}$ is the rate of increase of the energy of a magnetic tube of strength m, and Ω is the magnetomotive force along the tube. The proof of this equation is identical to the proof of equation (23).

585. Isolated equations. — All the above equations occur in pairs. Every electric or magnetic equation which involves, directly or indirectly, the conception of *electric conduction*, stands by itself, for there is no such thing as *magnetic conduction*. Thus all equations which involve electric current (conduction current), resistance, and electrostatic capacity, are isolated * equations.

* The two equations, $N = Li$ (296) and $Q = JE$ (229), are often given as a pair of electromagnetic equations, but they are analogous only in form. Physically, there is not the least similarity between them.

Remark. — All equations which involve the rate of change $\frac{dQ}{dt}$ of the strength of an electric tube, for example, equations (23), (27), (28), and (29), are identical to certain equations which involve *conduction current* instead of $\frac{dQ}{dt}$. Equations (23) and (29) of this chapter are identical to equations (219) and (274) respectively. An electric tube of which the strength is changing is, therefore, equivalent, in its magnetic action, to a conduction current. This variation of an electric tube is therefore called a *displacement current*, or a *dielectric current*.

586. Electromagnetic equations. — All phenomena which are associated in any way with the above equations are either wholly electric or wholly magnetic. The mutual dependence of electric and magnetic phenomena is a matter for distinct experimental discovery. There is a number of electromagnetic phenomena, and, any one of these phenomena being established by experiment, all the others can be shown to be necessary, as will be clearly seen in the following development. The phenomenon to be taken as fundamental is open to choice. We shall here choose the phenomenon of the production of e. m. f. around a closed loop by a changing magnetic flux through the loop, from which we get the following:

587. Independent electromagnetic equation.

$$E = -\frac{dM}{dt}. \tag{25}$$

That is to say, the e. m. f. e, around a closed loop (in a homogeneous medium) is proportional* to the rate of change $\frac{dM}{dt}$ of the magnetic flux through the curve. The minus sign is chosen for the reason that a positive value of $\frac{dM}{dt}$ gives a left-handed e. m. f. around a loop. (See Art. 533.)

* The proportionality factor, which is found to be the same for all homogeneous media, may be considered to be unity, inasmuch as we already have more electrical quantities (by one) than independent electric and magnetic equations, and it would be meaningless to introduce another which always has the same value.

588. Derived electromagnetic equations. — Consider an endless electric tube of strength Q linking with an endless magnetic tube of strength m (see Fig. 359). Let the positive directions around these tubes be chosen, as shown by the arrows. The electric flux through the tube Q is

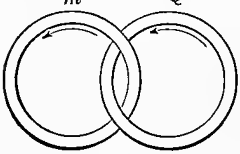

Fig. 359.

$$N = 4\pi Q \tag{a}$$

from equation (11); and the magnetic flux through the tube m is

$$M = 4\pi m. \tag{b}$$

The e. m. f.* around the electric tube is, by equation (25),

$$E = -\frac{dM}{dt}, \tag{25 bis}$$

or, using equation (b),

$$E = -4\pi\frac{dm}{dt}, \tag{26}$$

in which $\frac{dm}{dt}$ is the rate of change of the strength of the magnetic tube.

If the strength of the electric tube is changing, its energy will be changing at a rate $\frac{dW}{dt} = E\frac{dQ}{dt}$ from equation (23), or, substituting the value of E from (26), we have

$$\frac{dW}{dt} = -4\pi\frac{dm}{dt}\cdot\frac{dQ}{dt}. \tag{27}$$

* Since the e. m. f. around a closed curve is equal to the *total* $\left(-\frac{dM}{dt}\right)$ through it, it is allowable to consider each changing magnetic tube as producing its quota of the e. m. f. In the equations of this article, E is that *part* of the e. m. f. around the tube Q due to the action of the given magnetic tube m; $\frac{dW}{dt} = E\frac{dQ}{dt}$ is that *part* of the rate of change of energy of the electrical tube which depends upon the action of the given magnetic tube, etc.

Therefore, the product $\frac{dm}{dt} \cdot \frac{dQ}{dt}$ being positive, *the electric tube is losing energy at the rate* $4\pi\frac{dm}{dt} \cdot \frac{dQ}{dt}$. This loss of energy by the electric tube is dependent upon the mutual action of the electric and magnetic tubes, and independent of everything else, so that the magnetic tube must be *gaining energy at the rate* $4\pi\frac{dm}{dt} \cdot \frac{dQ}{dt}$; that is,

$$\frac{dW}{dt} = +4\pi\frac{dm}{dt} \cdot \frac{dQ}{dt}. \qquad (28)$$

Comparing this with equation (24), we see that $+4\pi\frac{dQ}{dt}$ is a m. m. f. (magnetomotive force) around the magnetic tube; that is,

$$\Omega = +4\pi\frac{dQ}{dt}, \qquad (29)$$

or, using equation (a),

$$\Omega = +\frac{dN}{dt}. \qquad (30)$$

Remark. — Equations (26) to (30) are derived from equation (25); any one of these equations being considered as a result of experiment may be used as an *independent* equation in place of equation (25). Equation (27) is best suited to this purpose because of its symmetry.

589. Transformation of equation (27). — The differentials $\frac{dm}{dt}$ and $\frac{dQ}{dt}$ in equation (27) render its use as an independent equation, along with equations (1), (2), (3), (4), (5), (6), (7), and (8), somewhat obscure. For this reason it is convenient to write equation (27) in a form which does not involve differentials as follows: Imagine the rate of change $\frac{dm}{dt}$ of the magnetic tube to be uniform, m being the total change in time t, then $\frac{dm}{dt} = \frac{m}{t}$. Similarly imagine the rate of change $\frac{dQ}{dt}$ of

the electric tube to be uniform, Q being the total change in time t, then $\frac{dQ}{dt} = \frac{Q}{t}$. Then $\frac{dW}{dt}$ will be uniform, and W being the whole energy transformed in time t, we have $\frac{dW}{dt} = \frac{W}{t}$, so that equation (27) becomes $\frac{W}{t} = -4\pi\frac{mQ}{t^2}$, or,

$$W = -4\pi\frac{mQ}{t}. \tag{33}$$

This equation expresses the fact that if a magnetic tube of strength m be formed around a dielectric current of strength $\frac{Q}{t}$ in the positive direction, then an amount of electric energy $4\pi\frac{mQ}{t}$ will be transformed into magnetic energy.*

590. Electromagnetic wave. — The equation (25) can be shown to be equivalent to

$$e = -\mu v f, \tag{31}$$

in which f is the intensity of a magnetic field, which is moving through space at a velocity v at right angles to itself, and e is an electric intensity at right angles to both v and f, produced by the moving magnetic field. Compare Art. 533, equation (290).

The equation (30) can be shown to be equivalent to

$$f = +kve, \tag{32}$$

in which e is the intensity of an electric field, which is moving through space at a velocity v at right angles to itself, and f is a magnetic intensity at right angles to both e and v, produced by the moving electric field.

Consider the case of two moving fields, e and f, such that

* Equation (33) is identical to the equation which expresses the work W done in carrying a magnetic pole of strength m around a conduction current of strength $\frac{Q}{t}$. (See Art. 508.)

each field is due to the motion of the other. Then equations (31) and (32) are simultaneous equations, and give

$$v^2 = -\frac{1}{\mu k}. \tag{33}$$

The velocity v is of course a vector, and its square is necessarily negative. Therefore ignoring the negative sign, we have

$$v = \frac{1}{\sqrt{\mu k}}. \tag{34}$$

Such mutually dependent moving electric and magnetic fields constitute an electromagnetic wave. The velocity of progression of such a wave is $\frac{1}{\sqrt{\mu k}}$ by equation (34). Various measurements by electrical methods of the quantity $\frac{1}{\sqrt{\mu k}}$ for air give for its value $298 \cdot 10^8 \frac{\text{cm}}{\text{sec}}$, which is *precisely the velocity of light in air.* Therefore, and for other reasons, it is thought that light consists of such electromagnetic waves. This is the so-called *electromagnetic theory of light.* The velocity of an electromagnetic wave, $v = \frac{1}{\sqrt{\mu k}}$, will be hereafter called simply the velocity of light.

591. Systems of electric and magnetic units. — We have the following * independent equations:

(1) $F = \frac{1}{k}\frac{Q_1 Q_{11}}{d^2}$, (3) $F = Qe$, (33) $W = 4\pi\frac{mQ}{t}$,

(4) $F = mf$ and (2) $F = \frac{1}{\mu}\frac{m_1 m_{11}}{d^2}$.

* Aside from such equations as $E \equiv \Sigma e \cdot \cos\epsilon \cdot \Delta s$, and aside from isolated equations. A given isolated equation is used to define the same electrical quantity in every case. For example, $Q = JE$ (229) defines electrostatic capacity J, when Q and E have been defined. The essential difference between the "electrostatic" and the "electromagnetic" systems of units is brought out most clearly by considering only those equations which are used to define different quantities in the respective systems.

These five independent equations contain the six electric and magnetic quantities K, Q, e, f, m, and μ. Therefore an arbitrary value must be assigned to some one of these quantities before the others are determined (or defined) by these equations.

592. "Electromagnetic" system of units. — The magnetic permeability μ of any given substance, say of air, may be taken as unity. Then,

$F = \frac{1}{\mu}\frac{m_1 m_{11}}{d^2}$, (2), defines strength of magnetic pole. (See Art. 331.)

$F = mf$, (4), defines intensity of magnetic field. (See Art. 336.)

$W = 4\pi\frac{mQ}{t}$, (33), defines electric current $\frac{Q}{t}$ or electric charge.*

$F = Qe$, (3), defines intensity of electric field. (See Art. 447.)

$F = \frac{1}{k}\frac{Q_1 Q_{11}}{d^2}$, (1), defines specific inductive capacity.

593. "Electrostatic" system of units. — The specific inductive capacity k of a given substance, say of air, may be taken as unity. Then,

$F = \frac{1}{k}\frac{Q_1 Q_{11}}{d^2}$, (1), defines electric charge (see Art. 446), electric current being defined as unit charge per second.

$F = Qe$, (3), defines intensity of electric field. (See Art. 447.)

$W = 4\pi\frac{mQ}{t}$, (33), defines strength of magnetic pole.

* The equation which expresses Ampere's Law, viz. $F = Ilf$ (199) may be derived from the equation $W = 4\pi\frac{mQ}{t}$, so that the definition of electric current by $W = 4\pi\frac{mQ}{t}$ is essentially identical to the definition given in Art. 353.

$F = mf$, (4), defines intensity of magnetic field. (See Art. 336.)

$F = \frac{1}{\mu}\frac{m_1 m_{11}}{d^2}$, (2), defines magnetic permeability.

Remark 1. — Resistance, electromotive force, magnetomotive force, electrostatic capacity, self-induction, etc., are defined by the same equations in both systems, in terms, of course, of μ, m, f, e, Q, and k, as defined above for the respective systems.

Remark 2. — The words "*electromagnetic*" and "*electrostatic*," as used to designate the two systems of electric and magnetic units, do not signify anything. The systems ought perhaps to be called Weber's and Faraday's systems respectively.

Proposition. — *The number of "electrostatic" units of charge in one "electromagnetic" unit of charge is equal to the velocity of light in air.*

Proof. — The velocity of light in air is $v = \frac{1}{\sqrt{k\mu}}$ by Art. 590. If an arbitrary value of unity be assigned to μ for air, then

$$v = \frac{1}{\sqrt{k}}. \tag{a}$$

Consider two equal charges Q. The force with which they repel at distance d is

$$F = \frac{1}{k}\frac{QQ}{d^2}, \tag{b}$$

or using $k = \frac{1}{v^2}$ from (a), we have

$$F = v^2\frac{QQ}{d^2}.$$

Let Q' be the same charge expressed in "electrostatic" units ($k = 1$ for air), then the force of repulsion, which is of course the same as before, is

$$F = \frac{Q'Q'}{d^2}. \tag{d}$$

Comparing (*c*) and (*d*), we have

$$v = \frac{Q'}{Q}. \tag{e}$$

That is, the number Q', which expresses a given charge in "electrostatic" units, is v times as large as the number Q, which expresses the same charge in "electromagnetic" units. Therefore the "electromagnetic" unit of charge is v times as large as the "electrostatic" unit of charge. Q.E.D.

Similarly it may be shown that there are v "electromagnetic" units of magnetic pole in one "electrostatic" unit of magnetic pole.

Remark. — The equation $W = QE$ (15) expresses the work W required to carry a given charge Q along a given path in a given electric field, E being the e. m. f. along the path. When the numerical value of Q (*given* charge) is large, the numerical value of E (given e. m. f.) must be correspondingly small, since the product QE is the same no matter what units are used in expressing Q and E. Therefore there are $\frac{1}{v}$ "electrostatic" units of e. m. f. in one "electromagnetic" unit of e. m. f.

The equation $Q = JE$ or $J = \frac{Q}{E}$ expresses the charge on a given condenser when charged with a given e. m. f. There are v "electrostatic" units of Q in one "electromagnetic" unit of Q, and $\frac{1}{v}$ "electrostatic" units of E in one "electromagnetic" unit of E, therefore there are v^2 "electrostatic" units of J in one "electromagnetic" unit of J.

The unit electric current is in every case a flow of one unit charge per second. Therefore there are v "electrostatic" units of current in one "electromagnetic" unit.

The equation $W = \frac{1}{2}Li^2$ expresses the kinetic energy W of a given current i in a given circuit of which the coefficient of self-induction is L. The product $\frac{1}{2}Li^2$ is therefore independent of the choice of electrical units, so that there must

be $\frac{1}{v^2}$ "electrostatic" units of L in one "electromagnetic" unit.

In this way the following relations are easily verified:

Number of "electrostatic" units in one "electromagnetic" unit.

For Q, I, Ω, $f \cdots v$.

For m, E, $e \cdots \frac{1}{v}$.

For J, k, and conductivity $\cdots v^2$.

For L, μ, and resistance $\cdots \frac{1}{v^2}$.

CHAPTER XVI.

ON THE MECHANICAL CONCEPTIONS OF ELECTRICITY AND MAGNETISM.*

594. Fundamental conception. — The ether is to be considered as built up of very small cells of two kinds, *positive* and *negative*, in such a way that only unlike cells are in contact. These cells are imagined to be so connected, where they are in contact, that if a cell be turned while the adjacent cells are kept stationary, then a torque, due to elastic reaction of adjoining cells, is brought to bear upon the turned cell, which tends to right it and which is proportional to the angle turned.

595. Conception of the magnetic field. — The ether cells at a point in a magnetic field are to be thought of as rotating about axes which are parallel to the direction of the field at the point, the angular velocity of the cells being proportional to the intensity of the field at the point. The positive cells rotate in the direction in which a right-handed screw would be turned that it might move in the direction of the field, and the negative cells rotate in the opposite direction. This opposite rotation of positive and negative cells is mechanically possible since only unlike cells are in contact.

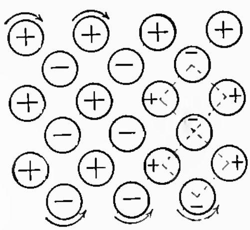

Fig. 360.

* Maxwell, Boltzmann, and others have demonstrated the mechanical possibility of a medium exhibiting the properties that are required of the luminiferous ether to explain the phenomena of electricity and magnetism and of light. The kinematics and dynamics of such a medium can be fairly stated only with the help of differential equations. The fundamental conceptions as here explained are approximations. A number of variations are possible in these conceptions.

This rotatory motion of the ether cells is represented in Fig. 360. The magnetic field is perpendicular to the plane of the paper and directed away from the reader; all the positive cells rotate clockwise, and the negative cells counter-clockwise. The kinetic energy per unit volume in such a system of rotating cells is proportional to the square of the angular velocity, which is consistent with the fact that the energy (kinetic) per unit volume in a magnetic field is proportional to the square of the intensity of the field.

596. Conception of the electric field. — The positive ether cells at a point in an electric field are to be thought of as displaced in the direction of the field, while the negative cells are displaced in the opposite direction; this displacement being proportional to the field intensity. Thus Fig. 361 represents the case in

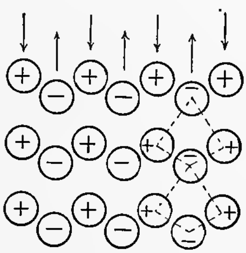

Fig. 361.

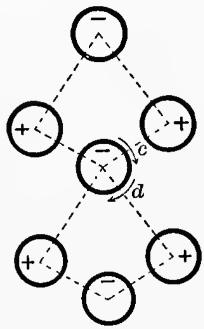

Fig. 362.

which the positive cells have been displaced towards the bottom of the page relatively to the negative cells. Figure 362 represents two meshes. The upward displacement of the positive cells has distorted these meshes, which are normally square. Since this cell-structure of the ether is elastic, as explained in Art. 594, its distortion as represented in Figs. 361 and 362 represents potential energy. The amount of potential energy per unit volume is proportional to the square of the displacement. This

is consistent with the fact that the energy (potential) per unit volume in an electric field is proportional to the square of the field intensity.

Remark. — The two negative cells to the right of the middle cell in Fig. 362 being displaced downwards, may be conceived to exert torques upon the middle cell as shown by the arrows c and d; which torques are proportional to the intensity of the electric field, *i.e.* to the displacements of the cells. The cells to the left exert equal but opposite torques upon the middle cell.

597. Explanation of induced electromotive force. — The e. m. f. around a closed curve is proportional to the rate of change of the magnetic flux.

Consider a closed boundary $ABCD$ (Fig. 363) in an electric field parallel to DA and CB. Let f be the intensity of the field along CB and f' the intensity along DA, and let l be the length of DA and of CB. The e. m. f. around the boundary $ABCD$ is $lf' - lf$. Now the total torque acting across BC on the enclosed cells is proportional to l and to f, and the total torque acting across AD is proportional to l and to f', but opposite in direction. Therefore the total torque acting to turn the enclosed cells is proportional to $lf' - lf$. The enclosed cells gain angular velocity under the action of this torque at a rate which is directly proportional to the torque, and inversely proportional to the number of cells which participate, so that the *product* of number of cells (area of $ABCD$) into angular acceleration (rate of change of magnetic field) is proportional to the torque or to the e. m. f. around $ABCD$, and this product is proportional to the rate of change of magnetic flux through $ABCD$. (See Art. 603 for explanation of induced e. m. f. due to moving magnetic field.)

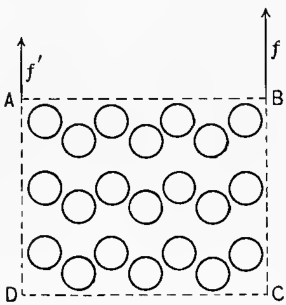

Fig. 363.

598. The energy stream in the electromagnetic field. — *Preliminary statement.* Consider three gear wheels A, B, C (Fig. 364). Let A and C exert equal and opposite torque actions upon B. Then if the wheels are turning, work will be transmitted from A to C or from C to A, according to direction of turning and to direction of torque action, and the rate of transmission of work will be proportional to the product of torque action into speed.

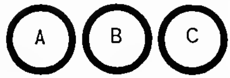

Fig. 364.

Imagine the cells in Fig. 361 to be rotating, positive cells in one direction, negative in the other, about axes perpendicular to the paper. This constitutes a magnetic field perpendicular to the electric field, which is towards the bottom of the page. On account of the torque actions between the cells, as explained in Art. 596, energy will be transferred to the right (or left) by each chain of geared cells at a rate which is proportional to the product of the intensity of the magnetic field into the intensity of the electric field, and the energy per second flowing across an area perpendicular to both electric field and magnetic field is proportional to the product of the respective field intensities into the area; for this area is proportional to the number of rows of cells which are acting as chains of gear wheels. The *energy stream, i.e. energy per unit area per second,* is therefore proportional to the product of magnetic and electric field intensities, and is at right angles to both. In case the electric and magnetic fields are not orthogonal, the energy stream is proportional to the vector part of this product.

599. The electric current. —Consider a wire AB (Fig. 365) along which an electric current is flowing. The magnetic field on opposite sides of AB is in opposite directions,

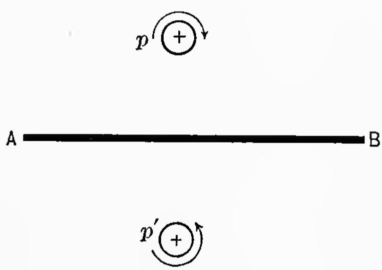

Fig. 365.

so that positive ether cells at p and p' are rotating in opposite directions, as shown in Fig. 365. Since an electric current may be maintained for an indefinite time, this opposite rotation of positive ether cells on the two sides of AB cannot be due to an ever-increasing ether distortion (the cells are geared together, as it were), but there must be a *slip* between adjacent cells somewhere between p and p'. This *slip* between adjacent ether cells (positive and negative cells) takes place in the material of the wire and constitutes the electric current.

600. Established electric currents flow in closed circuits. — Let AB (Fig. 366) be a wire carrying an established electric

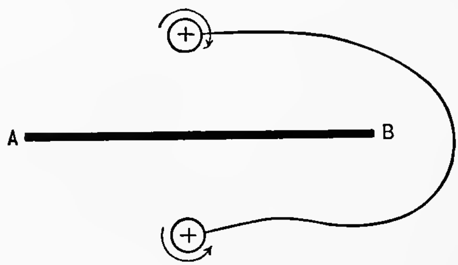

Fig. 366.

current. If this wire does not form a closed circuit, the opposite rotations of like ether cells on opposite sides of AB cannot continue without adjacent cells slipping on each other somewhere along any line passing around the end of AB.* That is, established *lines of slip* of the ether cells are necessarily closed lines. When a current does flow in a circuit which is not closed, an increasing ether distortion (electric field) is produced around the end portions of the circuit which produces (constitutes) electric charge there.

* This statement is inadequate unless the figure is thought to represent a flow of current in two dimensions; the complete statement for three dimensions is too complicated for present purposes.

601. Flow of energy in the neighborhood of an electric current. — Let Fig. 367 represent the neighborhood of a long wire *AB* through which electric current is flowing. The electric field in the neighborhood is parallel to the wire, and the magnetic field circles round the wire, as explained in Art. 349. The product of

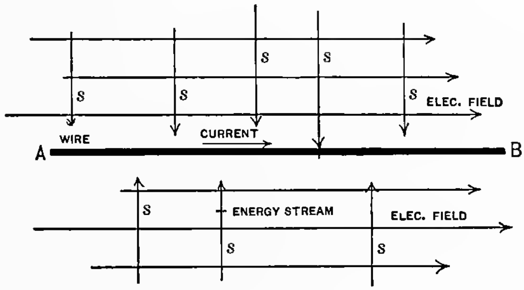

Fig. 367.

magnetic field intensity into electric field intensity is the energy stream, and this is directed towards the wire from all sides. This energy streaming in upon the wire changes into the heat which appears in the wire. In case the wire is of high resistance, the electric field (volts per centimeter) is of great intensity, and, for the same current and same intensity of magnetic field, the energy stream is correspondingly intense, making the wire very hot.

The student is referred to papers by J. H. Poynting, in the *Philosophical Transactions*, for full discussion of energy flow from a battery to the various parts of an electric circuit.

602. The charge on a condenser and its disappearance when the condenser plates are connected by a wire. — *Preliminary*

Fig. 368.

statement. Consider a row of gear wheels *A* to *B* (Fig. 368). If the wheel *B* is held stationary while *A* is turned in the direc-

tion of the arrow, the wheels will arrange themselves as shown in Fig. 369, alternate wheels being displaced upwards, and the intermediate wheels being displaced downwards. Conversely, a

Fig. 369.

number of geared wheels which by elastic action of any kind tend to stand in a straight row will be *relieved* from such a distortion as is represented in Fig. 369 by allowing freedom of rotation or *permitting slip anywhere across the row.*

Let A and B (Fig 370) be the plates of a charged condenser. The electric field f between these plates consists of upward

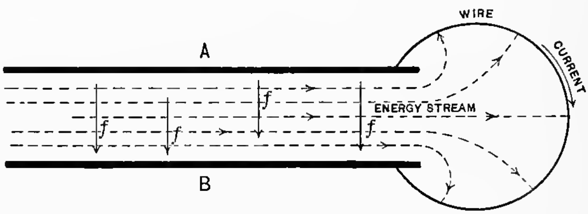

Fig. 370.

displacement of positive ether cells and downward displacement of negative ether cells.

The horizontal rows of cells are distorted as shown in Fig. 369, and if a wire is connected across from A to B anywhere, this will constitute a line of slip permitting the adjoining ether cells to rotate, and thus entirely relieving the distortion which constitutes the field between the plates. The potential energy of the ether distortion will be transmitted along the rows of geared cells, in the direction of the dotted arrows, to the wire, where it will appear as heat.

An electric spark is a line of slip produced by the breaking down of the mechanism which sustains the electric stress, and the manner of flow of electric energy in upon a spark is as here explained.

The explanation here given of the entire relief of the electric stress between two plates by the establishment of a conducting line (line of slip) between them applies equally to two adjacent oppositely charged bodies of any shape.

603. The production of an electric field by a moving magnetic field and the production of a magnetic field by a moving electric field; ether waves. — Imagine the ether cells between the dotted lines *A* and *B* (Fig. 371) to be rotating (magnetic field perpendicular to paper). There will soon be an elastic distortion (electric field towards top or bottom of page) between these rotating cells and the stationary cells to right and left. This elastic reaction will soon stop the motion of the cells between *A* and *B* and set the adjoining cells in motion. These will in turn act upon the cells next beyond, and so forth. Thus the original layer *AB* of the magnetic field will give rise to two waves, to right and left, and these waves will consist of mutually perpendicular electric and magnetic fields. If the cells between *A* and *B* (Fig. 371) are, to begin with, displaced upwards (figure represents positive cells only), their reaction upon adjoining cells will start rotatory motion, and the result will be two waves, to right and left, as before.

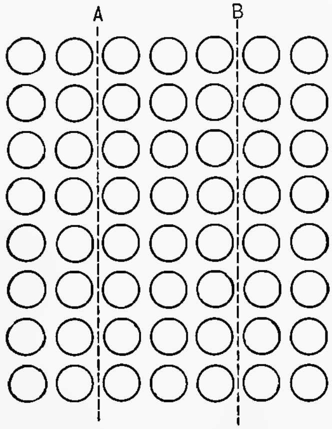

Fig. 371.

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